

1. Formulate the Kirchhoff relationship between the emissivity and the absorptive capacity. Emissivity function for perfect black body.
2. Estimate the emissivity for a perfect mirror (reflects all incident electromagnetic energy).
3. Specify the units of measurement for emissivity and absorption capacity.
4. How much energy (in Joules) does the human body radiate in 1 second? (body area  $2 \text{ m}^2$ , temperature  $36.6^\circ \text{C}$ )
5. In what region of the electromagnetic spectrum the maximum emissivity of the human body is located (temperature  $36.6 \text{ C}$ )?
6. Monochromatic light source emits 10 Watts of electromagnetic energy. How many photons does this source emit in one second if the wavelength of the electromagnetic wave is  $5000 \text{ \AA}$ ?
7. Monochromatic light source emits 1 Watt of electromagnetic energy. Estimate the mass loss of the source in one second due to the emission of photons, if the wavelength of the electromagnetic wave is  $400 \text{ nm}$ ?
8. The basic regularities of photo effect (With your comments).
9. The photoelectric effect is associated with the interaction of photons with electrons located on the surface of a metal (scattering of photons by an electron is similar to the Compton effect). But why does the energy of the emitted electrons not depend on the direction in which the photon hits the electron (unlike the photoelectric effect)?
10. Give a qualitative description of the Compton effect.
11. Calculate the **kinetic mass** and momentum for a photon with an wavelength of  $5000 \text{ \AA}$ .
12. At what frequency of the photon its energy equal to the rest energy of electron?
13. Calculate the kinetic part (associated with motion) of the total kinetic energy of an electron moving at a speed of  $0.9$  light speed.
14. What do the expressions for calculating the kinetic energy of an electron look like in the special theory of relativity and in classical mechanics?
15. Why is a hydrogen atom unstable from the point of view of classical physics?
16. Formulate Bohr's postulates for the hydrogen atom. Which postulate has no physical justification? Why?
17. The physical meaning of the Heisenberg uncertainty principle. Is this relationship somehow related to the limitations of the accuracy of physical measurements?
18. Physical interpretation and units of measurement of the wave function in cases 1d, 2d, and 3d.
19. Physical substantiation of the main properties of the wave function.
20. Prove the continuity equation for wave function. Probability current density.
21. Show that if the imaginary part of the wave function is equal to zero, then the probability current density for such a system is also equal to zero.
22. Why for the non-stationary Schrödinger equation (if the potential energy depends on time) it is impossible to divide the total wave function into parts depending on time and coordinates?
23. Show that the wave functions for one-dimensional motion of a free particle (use periodic boundary conditions) are orthonormal.
24. Give a physical interpretation of the positive and negative values of the quantum number for one-dimensional periodic motion of free particle.
25. Is it possible to calculate the exact value of the coordinate of a particle in states with quantum number  $n=1$ ? The motion is one-dimensional periodic and free.
26. Prove that kinetic energy operator is Hermitian.
27. Calculate the probability current density for periodic motion of free particle in one dimensional space.
28. Operators are commute or not commute :
  - a.)  $\hat{p}_x$  and  $\hat{p}_y$  ?
  - b.)  $\hat{p}_x^2$  and  $\hat{p}_y$  ?
  - c.)  $\hat{p}_x$  and  $\hat{y}$  ?

d.)  $\hat{p}_x$  and  $\hat{p}_y$  ?

e.)  $\hat{p}_x$  and  $\hat{x}^2$  ?

29. The angular momentum operator looks like this  $\hat{L} = \hat{r} \times \hat{P}$ . How does it look like operators  $\hat{L}_x, \hat{L}_y, \hat{L}_z$  ?

30. Do operators  $\hat{L}_x$  and  $\hat{x}$  commute?

31. Do operators  $\hat{L}_x$  and  $\hat{L}_y$  commute?

32. Is there a fundamental limitation on the lower limit of the error of physical measurements in classical physics?

33. How is the Heisenberg uncertainty principle related to the error of physical measurements?

For stepped barrier:

34. Show that for potential barrier for ( $E < U_0$ ) the flux of particles (probability density current), moving towards the barrier (incident particles) is  $j_i = \frac{\hbar k_1}{m} A^2$ .

35. Show that for potential barrier ( $E < U_0$ ) the flux for reflected particles (reflected from barrier) is

$$j_r = \frac{\hbar k_1}{m} B^2$$

36. Show that transmitted flux (inside of barrier for  $E < U_0$ ) is zero due to the fact that the wave function inside the barrier is a real function.

37. Physical meanings for reflection (R) and transmission (T) coefficients and its measurements units. How does it look like dependence of R and T on energy for stepped barrier in classical physics?

38. Calculate the energy dependencies of R(E) and T(E) for a stepped barrier in the case of  $E < U_0$ . Why don't particles penetrating the barrier and don't create a probability current density inside the barrier? What does the reflection process look like?

For rectangle barrier:

39. How does it look like Schrodinger equation for tunnel effect (topic 4.3) for regions 1, 2, and 3? Derive equations.

40. What does the Schrödinger equation and the continuity conditions look like for the tunneling effect in the case  $E > U_0$ ? Derive equations.

41. How does it look like dependence of R and T on energy for rectangle barrier in classical physics? What does it mean the tunnel effect for rectangle barrier?

42. What does the transparency window mean for a rectangular barrier and the conditions for its appearance?

43. What does the Schrödinger equation and the continuity conditions look like for the tunneling effect in the case  $E < U_0$ ? Derive equations.

For infinite potential well:

44. Can you show (just by logic and without any calculations) that the probability current density inside of the walls (areas 1 and 3) and inside the potential well (area 2) should be zero? Why?

45. Prove that the eigenfunctions for infinite potential form a system of orthonormal wave functions.

46. Calculate the average of the coordinate  $\langle x \rangle$ , the square of the coordinate  $\langle x^2 \rangle$ , and the square of the velocity  $\langle v^2 \rangle$ .

47. Why the solution for quantum number  $n=0$  must be ignored? Give a physical justification.

For finite potential well:

48. Can you show that equation for calculating energy of particle inside of **finite potential well**  $\tan\left(\frac{a\sqrt{2mE}}{\hbar}\right) = \frac{2\sqrt{E(U_0-E)}}{2E-U_0}$  give the correct values for energy for **infinite potential well** (topic 5.1)

$$E_n = \frac{(\pi \hbar)^2}{2Ma^2} \cdot n^2, \quad n=1, 2, \dots \text{ in limit } U_0 \rightarrow \infty ?$$

49. Why zero energy solution ( $E=0$ ) of equation  $\tan\left(\frac{a\sqrt{2mE}}{\hbar}\right) = \frac{2\sqrt{E(U_0-E)}}{2E-U_0}$  should be ignored?

Harmonic oscillator:

50. Why **asymptotical solution** of Schrodinger equation for harmonic oscillator is looks like so  $\psi(\xi) = e^{-\frac{\xi^2}{2}}$  ?

Why the option  $\psi(\xi) = e^{\frac{\xi^2}{2}}$  should be ignored?

51. Show that for **classical harmonic oscillator** average value of kinetic and potential energies are equal  $\langle E_{kin} \rangle = \langle E_{pot} \rangle$  .

PS! The classical expression for calculate average value of classical physical quantity is  $\langle A \rangle = \frac{1}{T} \int_0^T A(t) dt$

here T- averaging time.

52. Calculate the mean value of **x**-coordinate for harmonic oscillator  $\int_{-\infty}^{+\infty} \psi_n^* \hat{x} \psi_n dx$  . Tabular integral from lectures can be used. Results compare with the corresponding results for classical harmonic oscillator.

53. Calculate the mean value of momentum **p<sub>x</sub>** for harmonic oscillator  $\int_{-\infty}^{+\infty} \psi_n^* \hat{p}_x \psi_n dx$  . Tabular integrals from lectures can be used. Results compare with the corresponding results for classical harmonic oscillator.

54. Calculate the mean value of square **x<sup>2</sup>**-coordinate for harmonic oscillator  $\int_{-\infty}^{+\infty} \psi_n^* \hat{x}^2 \psi_n dx$  . Tabular integrals from lectures can be used. Results compare with the corresponding results for classical harmonic oscillator.

56. Calculate the next matrix element for **x**-coordinate for harmonic oscillator  $\int_{-\infty}^{+\infty} \psi_1^* \hat{x} \psi_3 dx$  . Tabular integrals from lectures can be used.

57. Calculate matrix element for the dipole transition from the ground state to the first excited state for harmonic oscillator  $\int_{-\infty}^{+\infty} \psi_0^* \hat{x} \psi_1 dx$  . Tabular integrals from lectures can be used.

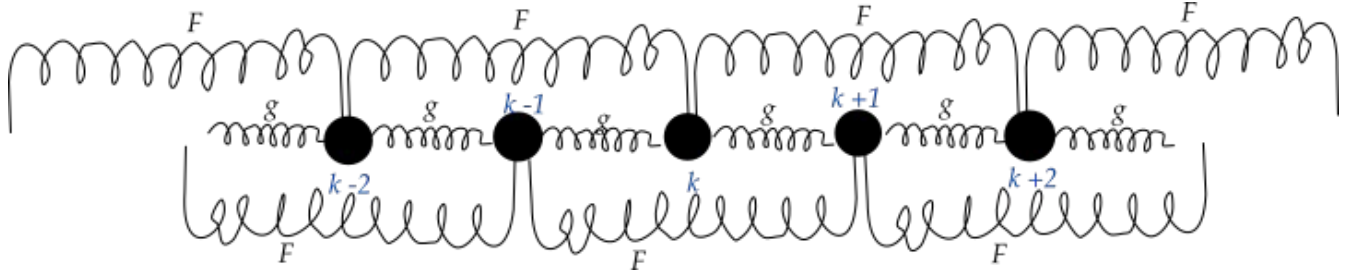
58. Why the power series  $v(\xi) = \sum_{r=0}^{+\infty} a_r \xi^r$  (here  $a_{r+2} = \frac{2r+1-\lambda}{(r+2)(r+1)} a_r$ ) for the wave function of a harmonic oscillator should be limited? Why is it necessary to take into account only a limited number of terms of series?

59. Calculate the mean value of the square of the coordinate (the value of the square of the deviation from the equilibrium position) for ground state of harmonic oscillator.

60. How can one calculate the energy of a harmonic oscillator at a non-zero temperature? (Don't forget that thermal motion is fully chaotic and can be described by a random force acting on a oscillating point mass).

Heat capacity for 1d chain of atoms:

61. How looks like the expression for potential energy of the 1d chain of atoms if we assume that each atom interacted not only with the first neighbors but with with the second neighbors too. Here  $\mathbf{g}$  and  $\mathbf{F}$  are force constant for different types of springs. It is enough to calculate the potential energy only for atom with number  $\mathbf{k}$ .



62. Can you proof that expression  $u_k(t) = \frac{1}{\sqrt{N}} \cdot \sum_{q=-\pi/a}^{+\pi/a} A_q \cdot e^{i(\omega t + qak)}$  is a solution of equation of corresponding equation of motion.

63. Give the physical meaning for density of vibration function  $g(\omega)$ . Calculate the value of next integral:  $\int_0^{\omega_0} g(\omega) d\omega = ?$  Here  $\omega_0 = \sqrt{\frac{4g}{m}}$ . What is the physical meaning of the calculation result?

<https://openstax.org/books/calculus-volume-1/pages/a-table-of-integrals> (integral 15 can help you).

64. Give a classical estimate of the heat capacity of a three-dimensional crystal (as was done for a one-dimensional lattice). Remember that atoms in a 3D crystal vibrate in all three dimensions it means that number of degrees of freedom must be increased (in compare with 1d lattice).

65. How the average value of the principal quantum number for harmonic oscillator depends on temperature. Please explain from a physical point of view the behavior of this function in the region of zero temperatures.

66. Why does the heat capacity tend to zero at low temperatures? Physical explanation.

67. Can you show that (for 1d crystal) **phase and group** velocities of harmonic waves are equal in limit of very long waves ( $\lambda \rightarrow \infty$ ).

68. Derive classical expressions for  $L_x, L_y, L_z$  projections for angular momentum vector.

69. Derive expressions for the projections of the angular momentum operator  $\hat{L}_x = -i\hbar \left( y \frac{\partial}{\partial z} - z \frac{\partial}{\partial y} \right)$ ,  $\hat{L}_y = -i\hbar \left( z \frac{\partial}{\partial x} - x \frac{\partial}{\partial z} \right)$ ,  $\hat{L}_z = -i\hbar \left( x \frac{\partial}{\partial y} - y \frac{\partial}{\partial x} \right)$ .

70. Derive the commutation relation for  $\mathbf{x}$  and  $\mathbf{y}$  projections of angular momentum operator  $[\hat{L}_x, \hat{L}_y] = i\hbar \hat{L}_z$ .

71. Is it possible measure simultaneously the  $\mathbf{x}$  and  $\mathbf{z}$  projections of angular momentum? Why? Proof.

72. Is it possible measure simultaneously square of angular momentum and its  $\mathbf{z}$  projections of angular momentum? Why? Proof.

73. Is it possible measure simultaneously square of angular momentum and its  $\mathbf{x}$  projection? Why? Proof.

74. Is it possible to measure simultaneously the absolute value of angular momentum  $|\vec{L}|$  and its  $x$  projection? Why? Proof.

75. Write an expression for the  $x$ ,  $y$  and  $z$  projection of circular frequency operators  $\hat{\omega}_x, \hat{\omega}_y, \hat{\omega}_z$  (The rotating body is a sphere with mass  $M$  and radius  $R$ ).

76. How can the angle between the angular momentum vector and the  $z$ -axis be calculated in quantum mechanics? Calculate the allowed possible values of this angle for orbital quantum number  $l=1$ .

77. How in quantum mechanics can be calculated the angle between the angular momentum vector  $\vec{L}$  and the  $z$ -axis? Calculate the values of this angle for **orbital quantum number**  $l=1$  and **magnetic quantum numbers**  $m=-2$  and  $+1$ .

78. How to calculate in quantum mechanics the kinetic energy of a rotating body with the moment of inertia  $I$ ? If the body is an electron moving around a nucleus in orbit with radius  $10^{-10}$  m. Calculate the velocity of the electron in this orbit if the orbital quantum number  $l=5$  (NB! Electron is a point particle).

79. How to calculate in quantum mechanics the kinetic energy of a rotating body with the moment of inertia  $I$ ? If the body is an electron moving around a nucleus in orbit with radius  $10^{-10}$  m. Calculate the minimum possible non-zero value of the velocity of the electron (NB! Electron is a point particle).

80. Can you prove the next expression  $[\hat{H}, \vec{L}^2]=0$ . What does it mean from a physical point of view?

81. Can you prove the next expression  $[\hat{H}, L_z]=0$ . What does it mean from a physical point of view?

82. Obtain the equation for the radial part of the wave function  $-\frac{\hbar^2}{2M} \left[ \frac{1}{r^2} \frac{d}{dr} \left( r^2 \frac{dR(r)}{dr} \right) \right] + \left[ \frac{\hbar^2 l(l+1)}{2M r^2} + U(r) \right] R(r) = E R(r)$

from the general Schrödinger equation for the hydrogen atom  $-\frac{\hbar^2}{2M} \left[ \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial}{\partial r} \right) - \frac{1}{\hbar^2 r^2} \vec{L}^2 - \frac{2M}{\hbar^2} U(r) \right] \psi(\vec{r}) = E \psi(\vec{r})$ .

83. What does the equation for the radial wave function look like if we assume that the electron is an uncharged particle?

84. What does the equation for the radial wave function look like if we assume that the electron is an uncharged particle and moves around the nucleus in an orbit with a fixed radius ( $r=\text{const}$ )?

85. Write the radial part of the wave function  $R_{nl}$  for quantum numbers  $n=3$  and  $l=2$ .

86. How does the radial part of the wave function look for 3s, 2d and 1f states?

87. How can we calculate the ionization energy of an electron from the ground state?

88. How can we calculate the ionization energy of an electron from the states 3s and 4p?

89. The total energy of an electron in a hydrogen atom can be calculated as follows:  $E_n = -R\hbar \frac{1}{n^2}$ . But how can we

calculate separately the kinetic and potential energy of an electron in a hydrogen atom for principal quantum number  $n=3$ ?

NB! The classical relation between the potential and kinetic energies for an electron in a hydrogen atom can be used.

90. Calculate the photon wavelength required to ionize a hydrogen atom from the ground state.

91. Calculate the possible maximum number of electrons in 1p and 3f orbitals for a hydrogen atom.

92. Calculate the possible maximum number of electrons in 4p and 2d orbitals for a hydrogen atom.

93. Calculate the number of electrons on the N-shell. Description of calculation.

94. Calculate the number of electrons on the O-shell. Description of calculation.

95. Calculate the number of electrons on the p-level. Description of calculation.

96. Calculate the number of electrons on the f-level. Description of calculation.

97. The electron configuration for a B atom looks like so:  $1s^2 2s^2 2p^1$ . What does it mean? Describe all numbers in the configuration description.

[https://www.chem.fsu.edu/chemlab/chm1045/e\\_config.html](https://www.chem.fsu.edu/chemlab/chm1045/e_config.html)

98. How looks like the electron configuration for O, Al and Li atoms? Why?

[https://www.chem.fsu.edu/chemlab/chm1045/e\\_config.html](https://www.chem.fsu.edu/chemlab/chm1045/e_config.html)

99. How looks like the electron configuration for N, Cl and K atoms? Why?

[https://www.chem.fsu.edu/chemlab/chm1045/e\\_config.html](https://www.chem.fsu.edu/chemlab/chm1045/e_config.html)

100. How looks like the electron configuration for Cu atom and  $\text{Cu}^{2+}$  ion? Why?

[https://www.chem.fsu.edu/chemlab/chm1045/e\\_config.html](https://www.chem.fsu.edu/chemlab/chm1045/e_config.html)

101. How looks like the electron configuration for Li, Na and K atoms? Why? What do all these materials have in common (in terms of physical properties)? How does this relate to the configuration of the electrons?

[https://www.chem.fsu.edu/chemlab/chm1045/e\\_config.html](https://www.chem.fsu.edu/chemlab/chm1045/e_config.html)

102. How looks like the electron configuration for Ne and Kr atoms? Why? What do all these materials have in common (in terms of physical properties)? How does this relate to the configuration of the electrons?

[https://www.chem.fsu.edu/chemlab/chm1045/e\\_config.html](https://www.chem.fsu.edu/chemlab/chm1045/e_config.html)

103. Why the vector of angular momentum and magnetic moment of electron are have an opposite directions?

104. What does the eigenvalue problem look like for the operator of square of the orbital magnetic moment  $\hat{\mu}_l^2$  of an electron in a hydrogen atom? Present the wavefunction and eigenvalues of this operator.

*Time independent perturbation theory (non degenerate case)*

105. Write the Schrodinger equation for **third order** approximation of perturbation theory.

106. Write the Schrodinger equation for **zero order** approximation of perturbation theory.

107. Can you prove the following expression (page 127): "In the first order approximation of  $\lambda$  coefficient  $a_n^1$  must satisfy  $a_n^1 + (a_n^1)^* = 0$ ".

108. Can you prove the following equation **for second order approximation** (page 129):  
$$\sum_{k \neq n} |a_k^1|^2 + ((a_n^2)^* + a_n^2) = 0$$

109. Can you prove that for **harmonic oscillator in constant force field** (page 129) "...the first order energy correction is equal to zero  $E_n^1 = H'_{nn} = 0$ ".

110. Why for the second-order energy correction of a harmonic oscillator in a constant force field we need to take into account only two terms of the sum  $n, n + 1$  and  $n, n-1$

$$E_n^2 = \sum_{k \neq n} \frac{|H'_{nk}|^2}{E_n^0 - E_k^0} = \frac{|H'_{n, n+1}|^2}{E_n^0 - E_{n+1}^0} + \frac{|H'_{n, n-1}|^2}{E_n^0 - E_{n-1}^0} = \frac{F^2}{\hbar \omega} (-|x_{n, n+1}|^2 + |x_{n, n-1}|^2) = -\frac{F^2}{2M\omega^2}.$$

111. Can you prove that for anharmonic oscillator the first order energy correction for cubic term must be equal zero (page 130)?

*Time independent perturbation theory (degenerate case)*

112. Task formulation. Initial non perturbed task. General representation of the Hamilton operator, wave function and energy for the perturbed problem.

113. First order perturbation theory equation. Secular equation. Calculation of the corrections for energy and wave functions.

114. Stark effect.

*Time dependent perturbation theory.*

115. Formulating of the problem for time dependent perturbed problem, write the Schrödinger equation and non-stationary wave function.

116. Equation for the time dependent expansion coefficients of the total wave function. Representation of the solution of this equation in the form of expansion into a series of perturbation theory approximations. The relationship of these coefficients and the probability of an interlevel transition.

117. Probability of interlevel transitions for first order of perturbation theory approximation. The case of harmonic external perturbation. The "golden rule" of quantum mechanics. Relationship between the "golden rule" and spectroscopy.

118. Calculation the probability of interlevel transition for harmonic oscillator and hydrogen atom in external electromagnetic wave by using "golden rule". Selection rules.

### *Relativistic quantum mechanics*

119. A. Einstein radiation theory. Induced and spontaneous transitions. Probabilities of transitions. Physical reason for spontaneous transitions. Spin-statistics theorem. Fermions and Bosons.

120. Klein-Gordon relativistic equation (Derivation).

121. Dirac equation. (Derivation) Pauli  $\sigma$  matrices.  $\gamma$ - representation of Dirac equation. Spinors and Bispinors.

122. Physical justification for the relationship between the Pauli spin matrices and the spin magnetic moment of fermions. Spin operator.