For infinite potential well:

1. Can you show (just by logic and without any calculations) that the probability current density inside of the walls (areas 1 and 3) and inside the potential well (area 2) should be zero? Why?

2. Prove that the eigenfunctions for infinite potential form a system of orthonormal wave functions.

3. Calculate the average of the coordinate $<x>$, the square of the coordinate $<x^2>$, and the square of the velocity $<v^2>$.

4. Why the solution for quantum number n=0 must be ignored? Give a physical justification.

For finite potential well:

5.Can you show that equation for calculating energy of particle inside of **finite potential well** $\tan\left(\frac{a\sqrt{2mE}}{\hbar}\right)=\frac{2\sqrt{E(U_0-E)}}{2E-U_0}$ give the correct values for energy for **infinite potential well** (topic 5.1) $E_n=\frac{(\pi\hbar)^2}{2Ma^2}\cdot n^2$, $n=1,2,\ldots$ in limit $U_0\to\infty$?

6. Why zero energy solution (E=0) of equation $\tan\left(\frac{a\sqrt{2mE}}{\hbar}\right)=\frac{2\sqrt{E(U_0-E)}}{2E-U_0}$ should be ignored?

Harmonic oscillator:

7. Why **asymptotical solution** of Schrodinger equation for harmonic oscillator is looks like so $\psi(\xi)=e^{-\frac{\xi^2}{2}}$?

Why the option $\psi(\xi)=e^{\frac{\xi^2}{2}}$ should be ignored?

8. Show that for **classical harmonic oscillator** average value of kinetic and potential energies are equal $<E_{kin}>=<E_{pot}>$.

PS! The classical expression for calculate average value of classical physical quantity is $<A>=\frac{1}{T}\int_0^T A(t)\,dt$

here T- averaging time.

9. Calculate the mean value of **x**-coordinate for harmonic oscillator $\int_{-\infty}^{+\infty}\psi_n^*\hat{x}\,\psi_n\,dx$. Tabular integral from lectures can be used. Results compare with the corresponding results for classical harmonic oscillator.

10. Calculate the mean value of momentum $\mathbf{p_x}$ for harmonic oscillator $\int_{-\infty}^{+\infty}\psi_n^*\hat{p}_x\,\psi_n\,dx$. Tabular integrals from lectures can be used. Results compare with the corresponding results for classical harmonic oscillator.

11. Calculate the mean value of square $\mathbf{x^2}$-coordinate for harmonic oscillator $\int_{-\infty}^{+\infty}\psi_n^*\hat{x}^2\,\psi_n\,dx$. Tabular integrals from lectures can be used. Results compare with the corresponding results for classical harmonic oscillator.

12. Calculate the next matrix element for **x**-coordinate for harmonic oscillator $\int_{-\infty}^{+\infty} \psi_1^* \hat{x}\, \psi_3\, dx$. Tabular integrals from lectures can be used.

13. Calculate matrix element for the dipole transition from the ground state to the first excited state for harmonic oscillator $\int_{-\infty}^{+\infty} \psi_0^* \hat{x}\, \psi_1\, dx$. Tabular integrals from lectures can be used.

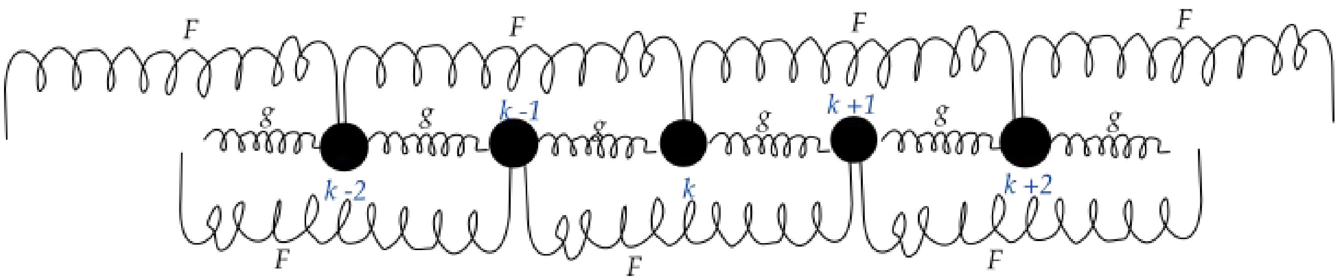
14. Why the power series $v(\xi)=\sum_{r=0} a_r \xi^r$ (here $a_{r+2}=\frac{2r+1-\lambda}{(r+2)(r+1)} a_r$) for the wave function of a harmonic oscillator should be limited? Why is it necessary to take into account only a limited number of terms of series?

15. Calculate the mean value of the square of the coordinate (the value of the square of the deviation from the equilibrium position) for ground state of harmonic oscillator.

16. How can one calculate the energy of a harmonic oscillator at a non-zero temperature? (Don't forget that thermal motion is fully chaotic and can be described by a random force acting on a oscillating point mass).

<u>Heat capacity for 1d chain of atoms:</u>

17. How looks like the expression for potential energy of the 1d chain of atoms if we assume that each atom interacted not only with the first neighbors but with with the second neighbors too. Here **g** and **F** are force constant for different types of springs. It is enough to calculate the potential energy only for atom with number **k**.



18. Can you proof that expression $u_k(t)=\frac{1}{\sqrt{N}}\cdot\sum_{q=-\pi/a}^{+\pi/a} A_q\cdot e^{i(\omega t+qak)}$ is a solution of equation of corresponding equation of motion.

19. Give the physical meaning for density of vibration function g(ω). Calculate the value of next integral: $\int_0^{\omega_0} g(\omega)\, d\omega = ?$ Here $\omega_0=\sqrt{\frac{4g}{m}}$. What is the physical meaning of the calculation result? https://openstax.org/books/calculus-volume-1/pages/a-table-of-integrals (integral 15 can help you).

20. Give a classical estimate of the heat capacity of a three-dimensional crystal (as was done for a one-dimensional lattice). Remember that atoms in a 3D crystal vibrate in all three dimensions it means that number of degrees of freedom must be increased (in compare with 1d lattice).

21. How the average value of the principal quantum number for harmonic oscillator depends on temperature. Please explain from a physical point of view the behavior of this function in the region of zero temperatures.

22. Why does the heat capacity tend to zero at low temperatures? Physical explanation.

23. Can you show that (for 1d crystal) **phase** and **group** velocities of harmonic waves are equal in limit of very long waves ($\lambda \to \infty$).