2) What does the Schrödinger equation and the continuity conditions looks like for the tunneling effect in the case $E>U_0$? Derive equations.

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Consider the next potential barrier: 97

$U_0,\ 0 \le x \le a$

$0,\ x<0, x>a$

Поснительный рисунок всегда необходим

$k'^2 = 2M(E - U_0) / \hbar^2$

$k^2 = 2ME / \hbar^2$

Region 1(U=0):

$\psi'' + k^2 \psi = 0$

$k^2 = 2ME / \hbar$

Solution:

$\psi_1 (x) = e^{(ikx)} + Be^{(-ikx)}$

Region 2($U=U_0$):

$\psi'' + k'^2 \psi = 0$

$k'^2 = 2M(E - U_0) / \hbar$

Solution:

$\psi_2 (x) = Ce^{(ik'x)} + De^{(-ik'x)}$

Region 3:

$\psi_3 (x) = Fe^{(ikx)}$

Continuity conditions:

$\psi_1 (0) = \psi_2 (0), \quad \psi_2 (x) = \psi_3 (x),$

$\psi'_1 (0) = \psi'_2 (0), \quad \psi'_2 (x) = \psi'_3 (x),$

$1+B = C+D, \quad Ce^{(ik'x)} + De^{(-ik'x)} = Fe^{(ikx)},$

$ik(1-B) = ik'(C-D), \quad k'(Ce^{(ik'x)} - De^{(-ik'x)}) = kFe^{(ikx)}$

13) Calculate the probability current density “j” for quantum number n=1.

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$\psi(x) = Ae^{ikx} + Be^{-ikx}$

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$k^2 = 2ME / \hbar^2$

Initial conditions are: $\psi_1(0) = \psi_1(x) = 0$, thus

$A + B = 0$, $Ae^{ikx} + Be^{-ikx} = 0$

$B = -A$, thus

$A(e^{ikx} + e^{-ikx}) = i2A\sin(ka) = 0$

$A \neq 0$, thus

$\sin(ka) = 0$

$ka = n\pi$, $n = 1,2,3,\ldots$

$k = n\pi/a$

Orthonormed wave functions are:

$\psi_n(x) = \sqrt{(2/a)}\sin(n\pi x/a)$

probability current density "j":

$j = \hbar/2mi(\psi^* d\psi/dx - \psi\, d\psi^*/dx)$.

Since $\psi_n(x)$ is real $\psi^* = \psi$

$j = 0$ for all n.

17) The wavefunction inside of finite potential well should be real or complex? Why? 19

If $E<U_0$ it would mean that particle is trapped inside a potential well. Wave function inside of it is real and oscillatory. Outside of well it decays exponentially, thus being real.

If $E>U_0$ time independent Schrödinger equation gives complex valued wave function

19) Why asymptotical solution of Schrodinger equation for harmonic oscillator is looks like so $\psi(\xi) = e^{-\xi^2/2}$? Why the option $\psi(\xi) = e^{\xi^2/2}$ should be ignored? 20

Consider potential energy of harmonic oscillator:

$U_x = M(wx)^2/2$

Schrödinger equation for a particle inside of it gives:

$-\hbar/(2M)\psi'' + M(wx)^2/(2)\,\psi = E\psi$

$\xi = x\sqrt{(Mw/\hbar)}$, $\qquad t = 2E/w\hbar$

$-\psi(\xi)'' + \xi^2\psi(\xi) = t\psi(\xi)$

Since the variable is not restricted we must find out whether there exist finite solutions if the variables tend to infinity. If $\xi \to \infty$, we demand that $\psi(\xi) \to 0$ (1)

If $|\xi| >> \lambda$ we have $-\psi(\xi)'' + \xi^2\psi(\xi) = 0$

One solution $\psi(\xi) = e^{(-\xi^2/2)}$ tends to zero (1),

While other $\psi(\xi) = e^{(\xi^2/2)}$ tends to infinity, which is why it should be ignored.

29) How looks like the expression for potential energy of the 1d chain of atoms if we assume that each atom interacted not only with the first neighbors but with with the second neighbors too. Here **g** and **F** are force constant for different types of springs. It is enough to calculate the potential energy only for atom with number **k**

$V = 0.5g\sum_k(u_k-u_{k-1})^2 + 0.5F\sum_k(u_k-u_{k-2})^2$

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