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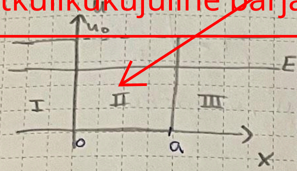
4) $E < U_0$

Vaatleme osakest, energiaga E, mis on joonisel potentsiaalbarjääriga

ristkülikukujuline barjäär, mitte kaev!



$$\begin{cases} \text{piirkond I} & x < 0 & U(x) = 0 \\ \text{piirkond II} & 0 \leq x \leq a & U(x) = U_0 \ (E < U_0) \\ \text{piirkond III} & x > a & U(x) = 0 \end{cases}$$

Kasutan statsionaarset Schrödingeri võrrandit, sest E ei sõltu ajast / on ühemõõtmeline

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kirjutatud mitu korda rohkem kui vaja. Milleks?

$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + U(x)\psi(x) = E\psi(x)$$

I piirkond $x < 0$

$$\frac{d^2\psi_1(x)}{dx^2} + \frac{2mE}{\hbar^2}\psi_1(x) = 0$$

üldlahend: $\psi_1(x) = Ae^{ikx} + Be^{-ikx}$, kus $k = \sqrt{\frac{2mE}{\hbar^2}}$

Ae^{ikx} – langev laine

Be^{ikx} – peegeldunud laine

II piirkond $0 \leq x \leq a$

$$\frac{d^2\psi_2(x)}{dx^2} - \frac{2m(U_0 - E)}{\hbar^2}\psi_2(x) = 0$$

üldlahend: $\psi_2(x) = Ce^{\alpha x} + De^{-\alpha x}$, $\alpha = \sqrt{\frac{2m(U_0 - E)}{\hbar^2}}$

These exponentional solutions characterize the evanescent wave inside the barrier

III piirkond $x > a$

$$\frac{d^2\psi_3(x)}{dx^2} + \frac{2mE}{\hbar^2}\psi_3(x) = 0$$

Üldlahend: $\psi_3(x) = Ee^{-ikx} + Fe^{ikx}$, $k = \sqrt{\frac{2mE}{\hbar^2}}$

Kuna laine tagasi ei peegeldu, siis $E = 0$

$$\psi_3(x) = Fe^{ikx}$$

Fe^{ikx} - edasi liikuv laine

Pidevuse tingimused

Füüsikaliselt kehtiva lahendi jaoks peavad nii lainefunktsioonid kui ka selle tuletis olema pidev rajapunktides

$x = 0$:

$$\psi_1(0) = \psi_2(0) \Rightarrow A + B = C + D \quad (1)$$

$$\psi_1'(0) = \psi_2'(0) \Rightarrow ik(A - B) = \alpha(C - D) \quad (2)$$

$x = a$:

$$\psi_2(a) = \psi_3(a)$$

$$Ce^{\alpha a} + De^{-\alpha a} = Fe^{ika} \quad (3)$$

$$\psi_2'(a) = \psi_3'(a)$$

$$\alpha(Ce^{\alpha a} - De^{-\alpha a}) = ikFe^{ika} \quad (4)$$

Transmission Coefficient Derivation

To find the transmission probability, we need to solve these four equations for the coefficients and calculate

$$T = \frac{|F|^2}{|A|^2}$$

$$\begin{cases} Ce^{\alpha a} + De^{-\alpha a} = Fe^{ika} \\ \alpha\left(Ce^{\alpha a} - De^{-\alpha a}\right) = ikFe^{ika} \end{cases}$$

$$2\alpha Ce^{\alpha a} = Fe^{ika}(\alpha + ik)$$

$$C = \frac{Fe^{ika}(\alpha + ik)}{2\alpha e^{\alpha a}} \Rightarrow D = \frac{Fe^{ika}(\alpha - ik)}{2\alpha e^{-\alpha a}}$$

Arendades need : $T = \dfrac{4k^2\alpha^2}{4k^2\alpha^2 + (k^2+\alpha^2)^2 \sinh^2(\alpha a)}$

Kõrgete barjääride jaoks : $\alpha a >> 1$

$$T \approx \frac{16k^2\alpha^2}{(k^2+\alpha^2)^2} e^{-2\alpha a}$$

sioomid

This exponential dependence on barrier width a is the hallmark of quantum tunneling and explains why the tunneling probability becomes extremely small for macroscopic objects or thick barriers

(13) $\psi(x) = Ae^{ikx} + Be^{-ikx}$, $k^2 = \dfrac{2ME}{\hbar^2}$ 15

Conditions $\psi_1(0) = \psi_1(x) = 0$, $\Rightarrow$

$\Rightarrow A + B = 0$, $Ae^{ikx} + Be^{-ikx} = 0 \Rightarrow A = -B$

$A\left(e^{ikx} + e^{-ikx}\right) = i2A\sin(ka) = 0$ $A = 0 \Rightarrow \sin(ka) = 0$

$ka = n\pi$, $n = 1, 2, 3 \ldots$ $k = \dfrac{n\pi}{a}$

e

Orthonormed wave functions are

$\boxed{\psi_n(x) = \sqrt{(2/a)}\sin\left(\frac{n\pi x}{a}\right)}$?????

$j = \dfrac{\hbar}{2m} i\left(\psi^*\left(\dfrac{d\psi}{dx} - \psi\dfrac{d\psi^*}{dx}\right)\right)$, $\psi_n(x)$ on reaalne $\psi^* = \psi$

$j = 0$ for all n

(16) for the case $E=0$:

5 punkti
raisatud tindi eest

Left side: if ~~E~~ $E=0$, we get $\tan\left(a\sqrt{\frac{2m(0)}{\hbar^2}}\right) = \tan(0) = 0$

Right side: $E=0 \Rightarrow 2\sqrt{0\cdot(U_0-0)} = 2\sqrt{0} = 0$

The ~~denom~~ denominator becomes $2(0) - U_0 = -U_0$

So the right side is: $\frac{0}{-U_0} = 0$

See on väga kummaline seletus.
Aga kui E=0, siis saab näidata, et lainefunktsioon on antud juhul null
(mis tähendab, et seda olekut ei eksisteeri).

Matemaatiliselt on mõlemad pooled nullid
but there are important physical reasons why this solution should be ignored.

1) Wavefunction behaviour: for $E=0$, the wavefunktions inside the well would have to be a straight line (or const linear function) which cannot satisfy both continuity conditions at the boundaries

2) Normalization issue: A particle with exactly zero energy would have an infinite de Broglie wavelenght, making proper normalization of the wavefunction impossible within a finite well

3) Physical reality: if $E=0$, a particle would have no kinetic energy inside the well and would not exhibit quantum confinement propertis

4) Mathematical singularity: As E approaches 0, the equation approaches a special case that requires careful limit analysis rather than direct substitution

5) Analytical: The transcendental equation was derived assuming $E \geq 0$, as the bound state energies of interest are positive values less than U_0

juht $E=0$ on piirjuht, mis jääb väljapoole transtsendentaalse võrrandi füüsikalise interpretatsiooni kehtivat vahemikku seotud olekute jaoks. Seetõttu ignoreerime lahendit $E=0$ puhul

26) Ühedimensionaalse ~~Schrodit~~ Schrödingeri võrrand harmoonilise ostsillaatori jaoks: 20

$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2}+\frac{1}{2}m\omega^2x^2\psi(x)=E\psi(x)$$

The wave function $\psi(x)$ can be expressed as a power series

$$\psi(\xi)=\sum_{n=0}^{\infty}a_n\xi^n, \quad \text{kus } \xi=\sqrt{\frac{m\omega}{\hbar}}\,x$$

The recurrence relation for the coefficients a_n is given by

$$a_{n+2}=\frac{2n+1-\lambda}{(n+2)(n+1)}a_n, \quad \lambda=\frac{2E}{\hbar\omega}$$

The power series solution should be limited for the following reasons:

1) Normalization: The wave function must be normalizable $\Rightarrow$ the total probability of finding the particle

must be finite. If the series is not limited, the wave function may diverge, leading to an infinite probability, which is meaningless

2) Physical boundaries: The harmonic oscillator potential grows infinitly as x increases. The wave function must decay to zero at large x to ensure the particle is confined within the potential well. Limiting the series ensures the wave function exhibit this decay

3) Eigenvalues and Eigenfunctions: The harmonic oscillator has discrete energy levels. The power series solution corresponds to these quantized energy levels. Limiting the series ensures wave function corresponding to specific E_n.

30 for atom with number k the equation of motion is

$$\ddot{U}_k = \frac{g}{m}\left(U_{k+1} - 2U_k + U_{k-1}\right) \qquad (1)$$

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for calculation the force action on the atom with number k we need to take into the account $\left(U_{k+1} - U_k\right)^2$ and $\left(U_k - U_{k-1}\right)^2$

The forces acting between the atoms are linear harmonic functions in exponential form

$$U_k(t) = \frac{1}{\sqrt{2mN}} A e^{i(\omega t + qak)} \quad (3)$$

m - mass of atom

N - total number of atoms

A - amplitude of given wave

ω - cycle frequency

q - wave vector $|q| = \frac{2\pi}{\lambda}$

λ - wave length

a - distance between nearest atoms

We have to take into account total set of all possible harmonic waves

$$U_k(t) = \frac{1}{\sqrt{N}} \sum_q A_q e^{i(\omega t + qak)} \quad (2)$$

q - index of different harmonic waves

If we substitute the solution (2) into the equation (1)

$$-\omega^2 A e^{i(\omega t + qak)} = \frac{g}{m}\left(A e^{i(\omega t + qa(k+1))} - 2A e^{i(\omega t + qak)} + A e^{i(\omega t + qa(k-1))}\right)$$

$$-\omega^2 e^{iqak} = \frac{g}{m}\left(e^{i(qa(k+1))} - 2e^{i(qak)} + e^{i(qa(k-1))}\right)$$

$$-\omega^2 = \frac{g}{m}\left(e^{iqa} - 2 + e^{-iqa}\right)$$

$$\omega^2 = \frac{2g}{m}\left(1 - \cos(qa)\right)$$

$$\omega^2 = \frac{4g}{m} \sin^2\left(\frac{qa}{2}\right), \quad \omega = \sqrt{\frac{4g}{m}}$$

$$\omega(q) = \omega_0 \left|\sin\left(\frac{qa}{2}\right)\right|$$

It is clear that $\omega(q) = \omega_0 \left|\sin\left(\frac{qa}{2}\right)\right| = \omega_0 \left|\sin\left(\frac{\left(q + \frac{2\pi n}{a}\right)a}{2}\right)\right| = \omega\left(q + \frac{2\pi n}{a}\right)$

For displacements (3) we have the same result:

$$u_k(t) = \frac{1}{\sqrt{N}} A e^{i(\omega t + qak)} = \frac{1}{\sqrt{N}} A e^{i\left(\omega t + \left(q \frac{2\pi n}{a}\right) ak\right)} = \frac{1}{\sqrt{N}} A e^{i(\omega t + qak)} \cdot$$

$$\cdot e^{i(2\pi nk)} \qquad n, k \in \mathbb{Z}$$

After applying $u_k = u_{k+N}$ in (3)

$$u_k(t) = \frac{1}{\sqrt{N}} A e^{i(\omega t + qak)} = \frac{1}{\sqrt{N}} A e^{i\omega t} \cdot e^{iqak} = \frac{1}{\sqrt{N}} A(t) e^{iqak}$$

$$e^{iqka} = e^{iq(k+N)a} \quad ; \quad e^{iqNa} = 1$$

This is possible if $qNa = 2\pi n$ and $q = \frac{2\pi}{Na} n = \frac{2\pi}{L} n$

L - total length of 1D crystal. If we take into account the range $\left[-\frac{\pi}{a} ; \frac{\pi}{a}\right] \Rightarrow q = \frac{2\pi}{L} n$, $n \in \left[\frac{-N}{2} ; \frac{N}{2}\right]$

q is discrete parameter which have N different values from $-\frac{\pi}{a}$ to $\frac{\pi}{a}$ with step $\Delta q = \frac{2\pi}{L}$

Conclusion

$$u_k(t) = \sum_{q=-\frac{\pi}{a}}^{\frac{\pi}{a}} \frac{1}{\sqrt{N}} A_q e^{i(\omega(q)t + qak)}$$