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20 7. Transmission probability for an electron on a rectangular barrier

An electron with kinetic energy $E = 8\,\mathrm{eV}$ is incident on a one-dimensional rectangular barrier of height $V_0 = 10\,\mathrm{eV}$ and width $a = 0.5\,A$. In the classically forbidden region ($E < V_0$), the wavefunction decays with decay constant

$$\kappa = \frac{\sqrt{2\,m\,(V_0 - E)\,(1.602 \times 10^{-19}\,\mathrm{J/eV})}}{\hbar}.$$

The transmission coefficient is

$$T = \frac{1}{1 + \dfrac{V_0^2}{4\,E\,(V_0 - E)}\,\sinh^2(\kappa\,a)}.$$

Using

$$m = 9.109 \times 10^{-31}\,\mathrm{kg}, \quad \hbar = 1.05457 \times 10^{-34}\,\mathrm{J\,s}, \quad 1\,\mathrm{eV} = 1.602 \times 10^{-19}\,\mathrm{J},$$

and

$$V_0 - E = 2\,\mathrm{eV}, \quad a = 0.5 \times 10^{-10}\,\mathrm{m},$$

$$\kappa \approx 7.25 \times 10^9\,\mathrm{m}^{-1}, \quad \kappa a \approx 0.3625, \quad \sinh^2(\kappa a) \approx 0.137.$$

And so,

$$T \approx \frac{1}{1 + \dfrac{100}{4 \cdot 8 \cdot 2} \times 0.137} = \frac{1}{1 + 0.2144} \approx 0.823,$$

the probability of transmission is about 82.3%.

20 13. Probability current density j for $n = 1$ in an infinite well

Wavefunctions in the infinite square well are

$$\psi_n(x) = \sqrt{\frac{2}{L}}\,\sin\!\left(\frac{n\pi x}{L}\right),$$

which are real. And so,

$$j = \frac{\hbar}{2mi}\big(\psi^* \, \partial_x \psi - \psi \, \partial_x \psi^*\big) = 0.$$

5 23. $\langle x^2 \rangle$ for quantum harmonic oscillator

The nth eigenstate of the quantum harmonic oscillator satisfies

$$\langle x^2 \rangle_n = \int_{-\infty}^{\infty} \psi_n^*(x) \, x^2 \, \psi_n(x) \, dx = \frac{\hbar}{2m\omega}(2n+1).$$

A classical oscillator with the energy $E_n = (n + \frac{1}{2})\hbar\omega$ has

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$$\langle x^2 \rangle_{\text{cl}} = \frac{E_n}{m\omega^2} = \frac{(n+\frac{1}{2})\hbar}{m\omega},$$

which agrees with quantum result in the large n limit, where $2n + 1 \approx 2n$.

20 35. Temperature dependence of n in a harmonic oscillator

In thermal equilibrium at temperature T,

$$P_n \propto e^{-E_n/(kT)}, \quad E_n = \big(n + \tfrac{1}{2}\big)\hbar\omega,$$

and

$$n = \frac{1}{e^{\hbar\omega/(kT)} - 1}.$$

As $T \to 0$, $\hbar\omega/(kT) \to \infty$, and so $\langle n \rangle \to 0$, and the oscillator occupies its plus zero-point energy.