85

20

6. Show that as U_0 tends to zero, the transfer coefficient T(E) tend to 1 for both cases $E>U_0$ and $E<U_0$.

(6) $x = \frac{E}{U_0}$ $\quad a^* = \frac{a\sqrt{2mU_0}}{\hbar}$

$E > U_0$:

$$T = \frac{4x(x-1)}{4x(x-1)\cos^2(a^*\sqrt{x-1}) + (2x-1)^2\sin^2(a^*\sqrt{x-1})}$$

$U_0 \to 0$: $\quad x = \frac{E}{U_0} \to \infty$, $\quad a^* \to 0$, sest $\sqrt{U_0} \to 0$

$\cos^2(a^*\sqrt{x-1}) \to 1 \qquad \sin^2(a^*\sqrt{x-1}) \to 0$

$$T = \frac{4x(x-1)}{4x(x-1)} = 1$$

$E < U_0$:

$$T = \frac{4x(1-x)}{4x(1-x)\cosh^2(a^*\sqrt{1-x}) + (2x-1)\sinh^2(a^*\sqrt{1-x})}$$

$U_0 \to 0$: $\quad a^* \to 0$

$\sqrt{1-x} = \sqrt{1-\frac{E}{U_0}} \to 0$, sest $E < U_0 \to 0$

$\cosh^2(a^*\sqrt{x-1}) \to 1 \qquad \sinh^2(a^*\sqrt{1-x}) \to 0$

$$T = \frac{4x(x-1)}{4x(x-1)} = 1$$

20 **12. What is the connection between the phenomenon of interference and the discreteness of the wave function and particle energy?**

Lõpmatus potentsiaalkaevus peab lainefunktsiooni väärtus seintel olema 0, sest osake ei saa barjääri tungida. Need nullid on seisulaine sõlmpunktid ja selleks peab lainefunktsioon iseendaga interfereeruma. Seisulained eksisteerivad ainult siis, kui kaevu laiusesse mahub täisarv n poollaineid L = n*(λ/2). See tingimus muudab lubatud lainepikkused λ diskreetseteks. Kuna lainepikkus on seotud impulsiga p =$\hbar$ /λ ja impulss omakorda energiaga E = p^2/2m, on ka impulss ja energia diskreetsed suurused.

10 **14. What does the Schrödinger equation and the continuity conditions looks like for finite potential well (for E<U₀)? Derive equations.**

(14) Lõplik potentsiaalauk $E < U_0$

$$U(x) = \begin{cases} 0, & \text{kui } 0 < x < a \quad (\text{sees}) \\ U_0, & \text{kui } x < 0,\ x > a \quad (\text{väljas}) \end{cases}$$

SEES: $-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} = E\psi \Rightarrow \frac{d^2\psi}{dx^2} + k^2\psi = 0$ $\qquad k = \frac{\sqrt{2mE}}{\hbar}$

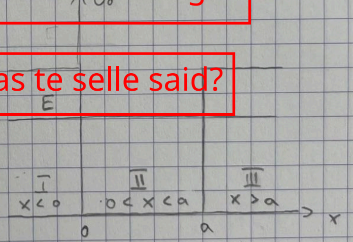
$\psi(x) = \boxed{A\sin(kx) + B\cos(kx)}$???? $< x < a$

VÄLJAS: $-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + U_0\psi = E\psi \Rightarrow \frac{d^2\psi}{dx^2} = k^2\psi$ $\qquad k = \frac{\sqrt{2m(U_0 - E)}}{\hbar}$

kus on liikme -kx -ga?

$x < 0 \qquad \psi(x) = \boxed{De^{kx}}$

$x > a \qquad \psi(x) = \boxed{Ce^{-k(x-a)}}$ Kuidas te selle said?



15 **24. Calculate the next matrix element for x-coordinate for harmonic oscillator $\int_{-\infty}^{+\infty}\psi^*_1 \hat{x}\psi_3 dx$. Tabular integrals from lectures can be used.**

(24) $\langle\psi_1|\hat{x}\psi_3\rangle = \int_{-\infty}^{+\infty}\psi_1(x)\, x\, \psi_3(x)\, dx = \int_{-\infty}^{+\infty}\psi_1^*(x)\,\hat{x}\,\psi_3(x)\, dx$

$\psi_n^* = \psi_n$, sest need on reaalsed funktsioonid. $\qquad \xi = \sqrt{\frac{m\omega}{\hbar}}\, x$

$$\int_{-\infty}^{+\infty}\psi_1(x)\, x\cdot\psi_3(x)\, dx = \int_{-\infty}^{+\infty} A_1 H_1(\xi)\, e^{-\frac{\xi^2}{2}}\cdot x\cdot A_3 H_3(\xi)\, e^{-\frac{\xi^2}{2}}\, dx =$$

$$= \boxed{A_1 A_2}\int_{-\infty}^{+\infty} H_1(\xi) H_3(\xi)\, x\cdot e^{-\xi^2}\, dx = \left[x = \sqrt{\frac{\hbar}{m\omega}}\,\xi \Rightarrow dx = \sqrt{\frac{\hbar}{m\omega}}\, d\xi\right] =$$

Kuidas saab nende parameetrite väärtusi arvutada?

$$= A_1 A_3\sqrt{\frac{\hbar}{m\omega}}\int_{-\infty}^{+\infty} H_1(\xi) H_3(\xi)\,\xi\cdot e^{-\xi^2}\, d\xi = (*)$$

$H_n(\xi) = (-1)^n e^{\xi^2}\cdot\frac{d^n}{d\xi^n}\left(e^{-\xi^2}\right)$ $\qquad H_1(\xi) = 2\xi$

$H_3(\xi) = 8\xi^3 - 12\xi$

(24) JÄTK

$H_1(\xi)H_3(\xi)\xi = 2\xi \cdot \xi \cdot (8\xi^3 - 12\xi) = 2\xi^2(8\xi^3 - 12\xi) =$

$= 16\xi^5 - 24\xi^3$

$(*) = A_1 A_3 \sqrt{\frac{\hbar}{m\omega}} \int_{-\infty}^{+\infty} (16\xi^5 - 24\xi^3) \cdot e^{-\xi^2} d\xi = 0$

Sest $\xi^5 e^{-\xi^2}$ ja $\xi e^{-\xi^2}$ on paaritud funktsioonid ja nende integraal on null.

20 **37. Can you show that (for 1d crystal) phase and group velocities of harmonic waves are equal in limit of very long waves ($\lambda\to\infty$).**

(37) Lähtudes dispersiooniseosest $\omega(q) = \omega_0 \cdot \left|\sin\left(q\frac{a}{2}\right)\right|$

Arendame siinuse väikeste q väärtuste korral (Taylori arendus)

$\sin\left(q\frac{a}{2}\right) \approx q\frac{a}{2}$ kui $q \to 0$

Seega $\omega(q) \approx \omega_0 \cdot q\frac{a}{2} = \left(\frac{\omega_0 a}{2}\right) q = v_0 q,$

Kui $\omega(q) \propto q$, siis $v_p = \frac{\omega(q)}{q} \approx v_0$

$v_g = \frac{d\omega}{dq} \approx v_0$

Seega on nii faasi kui grupikiirus võrdsed ja konstantsed.

Jah, ühe-dimensioonilises kristallis lähenevad nii faasikiirus $v_p = \omega/q$ kui ka grupikiirus $v_g = d\omega/dq$ samale väärtusele $v_0 = a_g\sqrt{(g/m)}$ kui $\lambda\to\infty$. Selle põhjuseks on asjaolu, et dispersiooniseos muutub selles piirväärtuses lineaarseks $\omega(q)\approx v0$, mille korrap ongi $v_p = v_g = v_0$.