

Töö on esitatud vales formaadis
(valemities), seega ainult 70

Kodutöö 2

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1. Give the physical interpretation and units of measurement of the wave function in cases 1d, 2d, and 3d .

A wave function is $\psi = \psi(r,t)$, its physical interpretation is the square modulus of the wave function $(|\psi(r,t)|^2)$ is proportional to the probability to find the particle in an small volume dV in at a given time t . $dP = |\psi(r,t)|^2 dV$. When we normalize the function we give it a unit of 1 because $\int |\psi(r,t)|^2 dV = 1$ because it's a probability and our particle does exist. Previous equations were already in 3D since we had dV instead of $dx dy$ and dz and to get a different dimension of 1D or 2D you have to adjust the equation and integral amount accordingly so 1D is 1 integral for dx and 2D is 2 integrals for dx and dy . Units for other dimension are 1D $(1/m^{3/2})$, 2D $(1/m)$ and 1D $(1/m^{1/2})$.

2. Eigenvalue problem for Hamilton operators is $H \psi_n = E_n \psi_n$ (here ψ_n is a set of orthonormal eigenfunctions for Hamiltonian operator H). How it looks like the eigenvalue E for eigenvalue problem $H \psi = E \psi$, here $\psi = \sum_n c_n \psi_n$. How coefficients c_n can be calculated? note: Then n is an index not a variable in equations

For the eigenvalue problem for the Hamiltonian operator is the equation $H \psi_n = E_n \psi_n$. To find the energy, you use an integral: $E_n = \int \psi_n^* H \psi_n dV$. To calculate the coefficients c_n you also integrate $c_n = \int \psi_n^* \psi dx$

- 3 17. The task for free one-dimensional motion of particle is stationary? Why? Write the corresponding Schrodinger equation for this task. Find the solution of it (wavefunction and energy of free particle). What gives the periodic boundary conditions (quantum number aka the physical state number)? Give the mathematical representation for it. Normalize the wavefunction. Calculate the probability density function. Is it depend on coordinate x , give the physical interpretation of the answer and relation one to Heisenberg uncertainty principle?

Firstly we take schrodingers equation for a single particle which is $H \psi = E_n \psi$, from that we get: $-\frac{(\hbar)^2}{2m} \frac{d^2 \psi}{dx^2} = E_n \psi$. Then we simplify with $k = (\sqrt{2mE_n})/\hbar$ getting $\frac{d^2 \psi}{dx^2} + k^2 \psi = 0$. From that we can get the wavefunction which is $\psi = c_1 e^{ikx} + c_2 e^{-ikx}$. The formula from Energy is also derived from the previous equations which is $E = \frac{(\hbar)^2 k^2}{2m}$. Moving on to the boundary conditions, taking a wall to which we have a distance of L with conditions of $\psi(x+L) = \psi(x)$. Taking our k from before and modifying it we can turn it into $k = \frac{2\pi n}{L}$ where we have brought in the quantum number n to find it we can go back to E where we slot in the new k and get $E_n = \frac{(2\pi n)^2 (\hbar)^2}{2m L^2}$ and from that $n = \frac{L}{2\pi \hbar} \sqrt{2mE}$. For the normalized wave function we use the function $\psi_n(x) = \frac{1}{\sqrt{L}} e^{ik_n x}$. Probability density function is from the square module of ψ and that is $1/L$.

- 4 31. How is the Heisenberg uncertainty principle related to the error of physical measurements?

Heisenbergs uncertainty principle links the position and momentum. $\Delta x \Delta p \geq \frac{\hbar}{2}$ and it links them together as follows: if Δp approaches 0 (meaning when know start to know its momentum with perfect accuracy) Δx approaches infinity (meaning we approach knowing nothing about the position) and vice versa.

5 36. What do the dependencies of transition and reflection coefficients on particle energy look like in the classical case? Why? 20

The two coefficients, R and L describe energies weather a particle gets reflected or passes trough the barrier. In a classical case if a particle doesn't have enough energy it gets reflected totally or when the energy is enough it totally passes through it. This happens because in classical physics there is no quantum tunneling and the probability is a paticle passing trough or being reflected is always 1 and 0 or vice versa while in quantum mechanics it is not so.