General properties of Schrodinger equation,operators and wave function.

1. Give the physical interpretation and units of measurement of the wave function in cases 1d, 2d, and 3d .

2. What properties must the wave function satisfy? Give the physical and mathematical substantiation.

3. **Derive** the continuity equation for wave function and give it a classical physical interpretation.

4. Show that in 3d-case the probability current density calculated by quantum equation $j_Q = \frac{i\hbar}{2m}\{\psi\hat{\nabla}\psi^* - \psi^*\hat{\nabla}\psi\}$ and it classical analog $j_C = n\cdot v$ have the same measurement units.

5. Probability current density. Physical interpretation of probability current density in classical physics . Show that if the wave function is real then the probability current density for such a system is equal to zero.

6. Prove that $\psi\hat{\Delta}\psi^* - \psi^*\hat{\Delta}\psi = \mathrm{div}\left(\psi\hat{\nabla}\psi^* - \psi^*\hat{\nabla}\psi\right)$.

7. How is looks like the solution of full Schrodinger equation $i\hbar\frac{\partial\psi}{\partial t} = \hat{H}\psi$ for stationary task (what is a stationary task?) ? What the different between this solution and solution for pure stationary Schrodinger equation $\hat{H}\psi = E\psi$?

8. Can you show that the average value for some physical quantity **A** (measurable value for given quantity) i.e. **<A>** do not depend on time for stationary solution of full Schrodinger equation $i\hbar\frac{\partial\psi}{\partial t} = \hat{H}\psi$?

9. The stationary Schrodinger equation (eigenvalue problem) for orthonormal wavefunctions ψ_1 and ψ_2 are follows $\hat{H}\psi_1 = E_1\psi_1$ and $\hat{H}\psi_2 = E_2\psi_2$. Here E_1 and E_2 are energies of system correspondingly for states 1 and 2. Calculate the energy for state describing by linear combination of these functions $\psi = c_1\psi_1 + c_2\psi_2$. What conditions must the coefficients c_1 and c_2 satisfy?

10. General expression for eigenvalue problem in quantum mechanics. How it is looks like for momentum operators p_x and kinetic energy operator ? The wavefunction in correspondig equations should be the same (same set of wave functions should be used)? Why? What is the physical meaning of eigenvalues for both eigenvalue problems? The eigenvalue is a complex number? Why?

11. Why in quantum mechanics should we use Hermitian operators to solve the eigenvalue problem? Definition of Hermite operators. Show that eigenvalue of Hermitian operators are real numbers.

12. Why in quantum mechanics should we use Hermitian operators to solve the eigenvalue problem? Definition of Hermite operators. Show that eigenfunctions of Hermitian operators form the set of orthonormal functions.

13. Prove that kinetic energy operator is Hermitian.

14. Prove that potential energy operator for harmonic oscillator is Hermitian.

15. Eigenvalue problem for Hamilton operators is $\hat{H}\psi_n = E_n \psi_n$ (here ψ_n is a set of orthonormal eigenfunctions for Hamiltonian operator $\hat{H}$). How is looks like the the eigenvalue **E** for eigenvalue problem $\hat{H}\phi = E\phi$, here $\phi = \sum_n c_n \psi_n$. How coefficients c_n can be calculated?

One dimensional motion of free particle.

16. The eigenvalue problem for x-projection of momentum operator is looks like so: $\hat{p}_x \psi_n = p_n \psi_n$. The average value of momentum generally for n-state can be calculated by this way: $<p>_n = \int \psi^* \hat{p}_x \psi dv$. Is it possible to say that always $<p>_n = p_n$, if not, so when is this possible and under what conditions?

17. The task for free one-dimensional motion of particle is stationary? Why? Write the corresponding Schrodinger equation for this task. Find the solution of it (wavefunction and energy of free particle). What gives the periodic boundary conditions (quantum number aka the physical state number)? Give the mathematical representation for it. Normalize the wavefunction. Calculate the probability density function. Is it depend on coordinate **x**, give the physical interpretation of the answer and relation one to Heisenberg uncertainty principle?

18. Prove that the wave functions for one-dimensional motion of a free particle (use periodic boundary conditions) are orthonormal. Write expression for wavefunction, energy and momentum for state number n=3.

19. Give a physical interpretation of the positive and negative values of the quantum number in one-dimensional periodic motion of free particle. Write expression for wavefunction, energy and momentum for state with number n=-2.

20. Is it possible to calculate the **exact** value of the coordinate of a particle in states with quantum number n=1? Why? What about the **average value** of coordinate x, is it possible to be calculated? (NB! The motion is one-dimensional periodic and free).

21. Calculate the probability density function for periodic motion of free particle in one-dimensional space. Is it possible to calculate exactly and simultaneously the momentum and coordinate fin this case? If not so what options for calculations of **x** and $\mathbf{p_x}$ are possible and why (give the explanation)? Relation to Heisenberg uncertainty principle.

22. Calculate the probability current density function for periodic motion of free particle in one-dimensional space. Give a physical interpretation of the result obtained.

23. Can you show (explain) that for a highly localized wave function (which allows one to precisely determine the position of a particle), the momentum of that particle cannot be accurately measured (calculated)? **NB! The movement of particles is free and one-dimensional.**

24. Calculate the **average** ($\langle x\rangle_n, \langle p\rangle_n$) and **exact** (x_n, p_n) **x**-coordinate and $\mathbf{p_x}$ momentum for a free particle in 1d (here $n \in Z$ is a quantum number of state). Were there any problems with the calculations of that four quantities? What kinds of problems? Why?

<u>Heisenberg uncertainty principle.</u>

25. Prove that for commuting operators the corresponding physical quantities can be exactly measured and calculated.

26. Heisenberg uncertainty principle. **Derive it!**

27. Operators are commute :

    a.) $\hat{p}_x$ and $\hat{p}_y$?

    b.) $\hat{p}_x^2$ and $\hat{p}_y$?

28. Operators are commute :

    a.) $\hat{p}_x$ and $\hat{y}$?

    b.) $\hat{p}_x$ and $\hat{p}_y$?

29. Operators are commute :

    a.) $\hat{p}_x$ and $\hat{p}_y$?

    b.) $\hat{p}_x$ and $\hat{x}^2$?

30. Is there a fundamental limitation on the lower limit of the error of physical measurements in classical physics?

31. How is the Heisenberg uncertainty principle related to the error of physical measurements?

Stepped barrier.

32. For stepped barrier: Show that for potential barrier for (E >U_0) the flux of particles (probability density current), moving towards the barrier (incident particles) is $j_i = \frac{\hbar k_1 A^2}{m}, \quad here\, k_1 = \frac{\sqrt{(2mE)}}{\hbar}$.

33. Show that for potential barrier (E > U_0) the flux for reflected particles (reflected from barrier) is $j_R = \frac{\hbar k_1 B^2}{m}, \quad here\, k_1 = \frac{\sqrt{(2mE)}}{\hbar}$

34. Proof that for reflection coefficient $\lim_{E \to +\infty} R(x) = 0$

35. Proof that for transmission coefficient $\lim_{E \to +\infty} T(x) = 1$

36. What do the dependencies of **transition** and **reflection** coefficients on particle energy look like in the classical case? Why?