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$$b) \hat{\nabla}(\psi \hat{\nabla} \psi^* - \psi^* \hat{\nabla} \psi) = \hat{\nabla} \psi \hat{\nabla} \psi^* + \psi \hat{\nabla} \psi^* - \hat{\nabla} \psi^* \hat{\nabla} \psi - \psi^* \hat{\nabla} \psi = \psi \hat{\nabla} \psi^* - \psi^* \hat{\nabla} \psi$$

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надо представлять не рукописный, а нормально оформленный вариант например в Libreoffice

12) We use them because eigen operators are real number quantities they express are real too.

Hermitean operator is an operator which equals its conjugated form

$$\hat{A}^\dagger = \hat{A}, \text{ thus}$$

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$$\langle \hat{A} \psi | \psi \rangle = \langle \psi | \hat{A} \psi \rangle$$

mis see on?

$$\langle \hat{A} \psi_m | \psi_n \rangle = \langle \psi_m | \hat{A} \psi_n \rangle = a_n \langle \psi_m | \psi_n \rangle = a_m^* \langle \psi_m | \psi_n \rangle$$

$$a_n^* = a_m$$

$$(a_n - a_m) \langle \psi_m | \psi_n \rangle = 0$$

$$\text{if } m \neq n \rightarrow$$

$$\langle \psi_m | \psi_n \rangle = 0$$

which is orthonormality definition

$$21) U=0$$

$$-\frac{\hbar^2}{2m} \frac{d^2 \psi}{dx^2} = E \psi$$

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$$\frac{d^2 \psi}{dx^2} = -k^2 \psi, \quad k = \frac{\sqrt{2mE}}{\hbar}$$

$$\psi = A e^{ikx} + B e^{-ikx}$$

$$\psi(x) = \psi(x+L)$$

$$\hat{\Delta}\psi = \left. \begin{aligned} A e^{ik(x+L)} + B e^{-ik(x+L)} &= A e^{ikx} + B e^{-ikx} \\ e^{ikL} + e^{-ikL} &= 1 \\ kL &= 2\pi n, \quad n=0, \dots, \infty \\ k_n &= \frac{2\pi n}{L} \end{aligned} \right\}$$

$$\psi(x) = A e^{ik_n x}, \quad B=0$$

$$\int_0^L |\psi(x)|^2 dx = 1$$

$$|A|^2 \int_0^L 1 dx = |A|^2 L = 1, \quad A = \frac{1}{\sqrt{L}}$$

Probability density:

$$|\psi_n(x)|^2 = \frac{1}{L}$$

It is not possible to calculate x and p_x exactly and simultaneously due to Heisenberg uncertainty principle:

$$\Delta x \cdot \Delta p_x \geq \frac{\hbar}{2}$$

But it is possible to find ^{their} expectation values.

$$\langle x \rangle = \int \psi^* [x] \psi dx$$

$$\langle p \rangle = \int \psi^* \left[-i\hbar \left(\frac{\partial}{\partial x} \right) \right] \psi dx$$

26) Assume $\langle A \rangle = \langle B \rangle = 0$, and

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$$[\hat{A}, \hat{B}] = i\hat{C}$$

$$\mathcal{J}(\lambda) = \int |(\lambda\hat{A} - i\hat{B})\psi|^2 dV \geq 0 \quad \lambda \in \mathbb{R}$$

$$\mathcal{J}(\lambda) = \int \psi^* (\lambda\hat{A} + i\hat{B})(\lambda\hat{A} - i\hat{B}) \psi dV =$$

$$\int \psi^* (\lambda^2 \hat{A}^2 + \lambda \hat{C} + \hat{B}^2) \psi dV$$

не понятен переход

$$\mathcal{J}(\lambda) = \lambda^2 \langle (\Delta A)^2 \rangle + \lambda \langle C \rangle + \langle (\Delta B)^2 \rangle$$

$$\mathcal{J}(\lambda) \geq 0$$

$$D = \langle C \rangle^2 - 4 \langle (\Delta A)^2 \rangle \langle (\Delta B)^2 \rangle$$

$$4 \langle (\Delta A)^2 \rangle \langle (\Delta B)^2 \rangle \geq \langle C \rangle^2$$

35)

$$T(x) = \frac{\kappa'}{\kappa} |C|^2, \quad \kappa' = \frac{\sqrt{2m(E-U)}}{\hbar}, \quad \kappa = \frac{\sqrt{2mE}}{\hbar}$$

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$$\lim_{E \rightarrow \infty} T(x) = \lim_{E \rightarrow \infty} \frac{\sqrt{2m(E-U)}}{2mE} |C|^2 = \lim_{E \rightarrow \infty} \sqrt{\frac{E-U}{E}} = 1$$

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