65

5. **Probability current density. Physical interpretation of probability current density in classical physics . Show that if the wave function is real then the probability current density for such a system is equal to zero.** 15

The probability current density is a mathematical quantity describing the flow of probability. Probability current density in classical physics represents the flow of power or energy per unit area per unit time, the direction of the probability current density vector indicates the direction in which energy is flowing through space. ?

If wave function is real: $\psi^*(r,t) = \psi(r,t)$

$$\vec{j} = \frac{i\hbar}{2m}\left(\psi \hat{\nabla} \psi^* - \psi^* \hat{\nabla} \psi\right) = \frac{i\hbar}{2m}\left(\psi \hat{\nabla} \psi - \psi \hat{\nabla} \psi\right) =$$

$$= \frac{i\hbar}{2m} \cdot 0 = 0$$

5

12. .**Why in quantum mechanics should we use Hermitian operators to solve the eigenvalue problem? Definition of Hermite operators. Show that eigenfunctions of Hermitian operators form the set of orthonormal functions.** pole definitsiooni

In quantum mechanics we should use Hermitian operators because Eigenvalues of Hermitean opertators are real numbers. Real eigenvalues ensure that the possible outcomes of measurements are real quantities, which aligns with experimental observations.
Hermitean operator is operator which equals to its conjugated operator (is therefore selfconjugated)

$2m$

$\hat{A} \varphi_n = a_n \varphi_n$ (hermiitilisus)

Eigenvalues are real numbers: $(a_n - a_m)\langle \varphi_m | \varphi_n \rangle = 0$ ei ole selge seletus

If $m \neq n$
$\langle \varphi_m | \varphi_n \rangle = 0 \Rightarrow$ different eigenfunctions are orthogonal

If $m = n$ $\langle \varphi_n | \varphi_n \rangle \neq 0 \Rightarrow \langle \varphi_n | \varphi_n \rangle = 1$

$\Downarrow$

Each eigenfunction is normalized to 1

Conclusion: $\langle \varphi_m | \varphi_n \rangle = \delta_{nm}$

21. **Calculate the probability density function for periodic motion of free particle in one-dimensional space. Is it possible to calculate exactly and simultaneously the momentum and coordinate fin this case? If not so what options for calculations of x and px are possible and why (give the explanation)? Relation to Heisenberg uncertainty principle.** 10

$$\hat{H}\cdot\varphi = E\varphi$$

$$\hat{H} = \frac{\hat{p}_x^2}{2m} = \frac{-\hbar^2}{2m}\frac{d^2}{dx^2}$$

$$\frac{-\hbar^2}{2m}\frac{d^2\varphi}{dx^2} = E\varphi \qquad \frac{d^2\varphi}{dx^2} = \underbrace{-\frac{2mE}{\hbar^2}}_{=k^2}\varphi$$

$$\frac{d^2\varphi}{dx^2} = -k^2\varphi$$

$$\varphi'' = -k^2\varphi$$

$$\lambda^2 + k^2 = 0$$

$$\lambda_{1,2} = \mp ik$$

$$\varphi(x) = c_1 e^{ikx} + c_2 e^{-ikx}$$

= eeldan, et liigub pos suunas

$$\varphi(x) = c_1 e^{ikx}$$

$$\int_{+\infty}^{+\infty} \varphi^*(x)\,\varphi(x)\,dx = 1$$

$$c^2\int_{-\infty}^{+\infty} 1\cdot dx = 1$$

$$\varphi(x) = \varphi(x+L)$$

$$c_1 e^{ikx} = c^{ik(x+L)}$$

$$e^{ikx} = 1$$

$$\int_0^L \varphi_n^*(x)\,\varphi(x)\,dx = 1 \Rightarrow c = \frac{1}{\sqrt{L}}$$

$$\boxed{\varphi_n(x) = \frac{1}{\sqrt{L}}e^{ikx}}$$

see on lainefunktsioon aga mitte probability density

Due to the principles of wave-particle duality and the uncertainty principle, it is not possible to simultaneously determine the exact values of both momentum (pxp_xpx) and position (xxx) of a particle with arbitrary precision.

This limitation is related to the wave nature of particles and is described by the Heisenberg uncertainty principle.This principle implies that the more precisely you know the position of a particle , the less precisely you can know its momentum. This inherent uncertainty arises due to the wave-like nature of particles, where their position and momentum are represented by wavefunctions that are related through Fourier transforms

Options for calculating x and px
In practice, when measuring the position xxx of a particle, the measurement disturbs the momentum pxp_xpx, and vice versa. This disturbance arises because measuring one observable affects the wavefunction describing the other observable, due to the non-commutative nature of position and momentum operators in quantum mechanics.

25. **Prove that for commuting operators the corresponding physical quantities can be exactly measured and calculated.** 20

$$[\hat{A}, \hat{B}]\varphi = 0$$

$\hat{A}\varphi = a\varphi$ & $\hat{B}$; $\hat{B}\hat{A}\varphi = a\hat{B}\varphi = ab\varphi$

$\hat{B}\varphi = b\varphi$ & $\hat{A}$; $\hat{A}\hat{B}\varphi = b\hat{A}\varphi = ba\varphi$

$$\hat{B}\hat{A}\varphi - \hat{A}\hat{B}\varphi = 0$$

$(\hat{B}\hat{A} - \hat{A}\hat{B})\varphi = 0$ $\Rightarrow$ commute

$\Downarrow$

both operators can be simultaneously diagonalized

~~Example:~~

$\Downarrow$

corresponding physical quantities $(\hat{A} ; \hat{B})$ can be exactly measured and calculated

Heisenbergi määramatuse printsiip

35. 5. Proof that for transmission coefficient lim E→+∞ T(x)=1 15

$$\lim_{E \to +\infty} T(x) = 1$$

$$T(x) = \frac{4\sqrt{x} \cdot \sqrt{x-1}}{\left(\sqrt{x} + \sqrt{x-1}\right)^2} \qquad x = \frac{E}{U_0}$$

tõsiselt? miks?

$$\lim_{E \to +\infty} \frac{4\sqrt{x}\,\sqrt{x-1}}{\left(\sqrt{x} + \sqrt{x-1}\right)^2} \;\boxed{\sqrt{x-1} \approx \sqrt{x}}\; \frac{4\sqrt{x} \cdot \sqrt{x}}{\left(\sqrt{x} + \sqrt{x}\right)^2} = \frac{\cancel{4x}^{\,1}}{\cancel{4x}_{\,1}} = 1$$