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Quantum mechanics IV

Karen Dunaway, 233616YAFB

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Question 6: Zeeman Effect. Derive a correction of energy and wavefunction.

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The Zeeman effect describes how atomic energy levels split under the influence of an static magnetic field $\mathbf{B}$. The total Hamiltonian:

$$\hat{H} = \hat{H}_0 + \hat{H}' \quad \text{with} \quad \hat{H}' = -\mu \cdot \mathbf{B}$$

For orbital angular momentum, the magnetic moment of an electron:

$$\mu = -\frac{e}{2m}\hat{\mathbf{L}} \quad \Rightarrow \quad \hat{H}' = \frac{e}{2m}\hat{\mathbf{L}} \cdot \mathbf{B}$$

Assuming $\mathbf{B} = B\hat{z}$, we get:

$$\hat{H}' = \frac{eB}{2m}\hat{L}_z$$

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First-order Energy Correction

$$E^{(1)} = \boxed{\langle n, l, m|\hat{H}'|n, l, m\rangle} = \frac{eB}{2m}\hbar m$$

So the perturbed energy becomes:

$$E = E_n^0 + \frac{e\hbar B}{2m}m$$

First-order Wavefunction Correction

The first-order correction to the wavefunction is:

$$\left|\psi^{(1)}\right\rangle = \sum_{k \neq n} \frac{\langle k|\hat{H}'|n\rangle}{E_n^0 - E_k^0}|k\rangle$$

In this case, $\hat{H}' \propto \hat{L}_z$, so corrections involve states with the same n, l but different m.

Question 7: Time-dependent Schrödinger Equation

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(a) $\hat{H}_0 \varphi_n^0 = E_n^0 \varphi_n^0$

(b) $i\hbar \frac{d}{dt} \psi_n^0 = \hat{H}_0 \psi_n^0$

How is looks like the solution for (b) in the stationary case? What is the relation between wavefunctions φ_n^0 and ψ_n^0? What is the representation of the wavefunction in the case of a time-dependent perturbation:

$$i\hbar \frac{d}{dt} \psi_n(t) = \left(\hat{H}_0 + \hat{H}'(t) \right) \psi_n(t)$$

Stationary Case Solution

$$\psi_n^0(t) = \varphi_n^0 \cdot e^{-iE_n^0 t/\hbar}$$

This shows that the full wavefunction ψ_n^0 is just the stationary eigenfunction φ_n^0 with a time-dependent phase.

Time-Dependent Perturbation

When the perturbation $\hat{H}'(t)$ is time-dependent, we expand the solution as:

$$\psi_n(t) = \sum_k c_k(t) \varphi_k^0 e^{-iE_k^0 t/\hbar}$$

This turns the Schrödinger equation into a system of differential equations for the time-dependent coefficients $c_k(t)$, which can be solved using time-dependent perturbation theory.

Question 13. What is the meaning of the theorem on the relationship between spin and statistics? 20

The spin statistics theorem is a theory that establishes a connection between the intrisic spin of particles and their quantum statistics: It says that particles with integer spin obey Bose-Einstein statistics and are called bosons. It is said that their wavefunctions are symmetric under particle exchange, and they can occupy the same quantum state.

$$f(E,T) = \frac{1}{e^{E/kT} - 1}$$

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On the other hand particles with half-integer spin obey Fermi-Dirac statisticsand are called fermions. Their wavefunctions are antisymmetric under particle exchange and follow the Pauli exclusion principle. Which means no 2 fermions can occupy the same quantum state

$$f(E,T) = \frac{1}{e^{(E-\mu)/kT} + 1}$$