20

4. Derive the system of linear equations for first order corrections for energy and zero order correction for wave - function. Secular equation.

We consider a quantum system with a perturbed Hamiltonian

$$\hat{H} = \hat{H}_0 + \lambda \hat{H}',$$

where $\hat{H}_0$ has a degenerate eigenvalue $E^{(0)}$ corresponding to a degenerate subspace spanned by unperturbed eigenfunctions $\{\psi_k^{(0)}\}$.

We expand the energy and wavefunction in powers of λ as

$$E = E^{(0)} + \lambda E^{(1)} + \dots, \quad \psi = \psi^{(0)} + \lambda \psi^{(1)} + \dots$$

Substituting into the time-independent Schrödinger equation

$$\hat{H}\psi = E\psi,$$

and collecting terms of order λ, we get

$$(\hat{H}_0 - E^{(0)})\psi^{(1)} + (\hat{H}' - E^{(1)})\psi^{(0)} = 0.$$

Projecting onto the degenerate basis $\{\psi_n^{(0)}\}$, we obtain:

$$\left\langle \psi_n^{(0)} \middle| (\hat{H}' - E^{(1)}) \middle| \psi^{(0)} \right\rangle = 0.$$

Assuming the corrected wavefunction is a linear combination of the degenerate states:

$$\psi^{(0)} = \sum_k c_k \psi_k^{(0)},$$

the projection becomes:

$$\boxed{\sum_k \left(H'_{nk} - E^{(1)} \delta_{nk} \right) c_k = 0,}$$ Kuidas see võrrandi saadi?

where $H'_{nk} = \left\langle \psi_n^{(0)} \middle| \hat{H}' \middle| \psi_k^{(0)} \right\rangle$.

This gives a homogeneous system of linear equations:

$$\sum_k \left(H'_{nk} - E^{(1)} \delta_{nk} \right) c_k = 0.$$

To have non-trivial solutions, the determinant must vanish, giving the secular equation:

$$\det \left(H'_{nk} - E^{(1)} \delta_{nk} \right) = 0.$$

Solving this gives the allowed first-order corrections $E^{(1)}$ to the energy. The corresponding eigenvectors $\vec{c}$ give the linear combination coefficients for the corrected wavefunctions.

30

7. For equations

$$\text{a)} \quad \hat{H}_0 \varphi_n^0 = E_n^0 \varphi_n^0$$

$$\text{b)} \quad i\hbar \frac{d}{dt} \psi_n^0 = \hat{H}_0 \psi_n^0$$

how does the solution for equation b) look in the stationary case? What about the relation between wavefunctions φ_n^0 and ψ_n^0?
What about the representation of the wave function in the case of a time-dependent perturbation

$$i\hbar \frac{d}{dt} \psi_n = \left(\hat{H}_0 + \hat{H}'(t) \right) \psi_n \, ?$$

Stationary Solution

In the stationary case, the solution to equation (b) has the form:

$$\psi_n^0(t) = \varphi_n^0 \, e^{-iE_n^0 t/\hbar}$$

This shows that **the relation between the two is:** $\psi_n^0(t)$ is the time-dependent wavefunction corresponding to the stationary state φ_n^0, multiplied by a time-dependent phase factor.

Time-Dependent Perturbation

Time-dependent perturbation:

$$i\hbar \frac{d}{dt} \psi_n(t) = \left(\hat{H}_0 + \hat{H}'(t) \right) \psi_n(t),$$

we represent $\psi_n(t)$ in terms of the unperturbed eigenfunctions:

$$\psi_n(t) = \sum_k c_k(t) \varphi_k^0 \, e^{-iE_k^0 t/\hbar}$$

Substituting this into the Schrödinger equation leads to a system of differential equations for the time-dependent coefficients $c_k(t)$:

$$i\hbar \frac{dc_m(t)}{dt} = \sum_n H'_{mn}(t) c_n(t) e^{i(E_m^0 - E_n^0)t/\hbar},$$

where

$$H'_{mn}(t) = \varphi_m^0 \hat{H}'(t) \varphi_n^0$$

These equations govern the transitions between energy levels due to the time-dependent perturbation.

20

15. Difference Between Schrödinger and Dirac Equations

1. Schrödinger Equation

$$i\hbar\frac{\partial}{\partial t}\psi(\vec{r},t) = \left(-\frac{\hbar^2}{2m}\nabla^2 + V(\vec{r})\right)\psi(\vec{r},t)$$

2. Dirac Equation

$$i\hbar\frac{\partial}{\partial t}\Psi(\vec{r},t) = \left[-i\hbar c\,\vec{\alpha}\cdot\nabla + \beta mc^2\right]\Psi(\vec{r},t)$$

where

- $\Psi(\vec{r},t)$ — Dirac spinor wavefunction, a 4-component object representing spin-$\frac{1}{2}$ particles (like electrons)
- $\vec{\alpha}$ — Vector of Dirac matrices that couple spatial derivatives to spinor components
- β — Dirac matrix related to rest energy
- c — Speed of light

Differences

- Schrödinger is non-relativistic, Dirac is relativistic.
- Schrödinger uses a scalar wavefunction ψ, Dirac uses a four-component spinor Ψ.
- Schrödinger does not include intrinsic spin, Dirac includes spin-$\frac{1}{2}$ as a built-in feature.
- Schrödinger does not predict the existence of antimatter, Dirac does.
- Dirac accounts for relativistic rest energy mc^2.
- Schrödinger is used for low speed, non-relativistic quantum systems, Dirac is used for high speed, relativistic spin-$\frac{1}{2}$ particles like electrons.