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2. Write the general Schrodinger equation for perturbation theory in degenerate case. Helping parameter.

In degenerate perturbation theory, the total Hamiltonian is written as:

$$\hat{H} = \hat{H}_0 + \lambda \hat{H}'$$

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where:

- $\hat{H}_0$ is the unperturbed Hamiltonian (with degenerate eigenstates),
- $\hat{H}'$ is the perturbation Hamiltonian,
- λ is a small dimensionless parameter (helping parameter).

The time-independent Schrödinger equation is:

$$\hat{H}\Psi_n = E_n \Psi_n$$

Expanding the energy and wavefunction in powers of λ:

$$E_n = E_n^{(0)} + \lambda E_n^{(1)} + \lambda^2 E_n^{(2)} + \cdots$$

$$\Psi_n = \Psi_n^{(0)} + \lambda \Psi_n^{(1)} + \lambda^2 \Psi_n^{(2)} + \cdots$$

In degenerate perturbation theory, one typically diagonalizes the perturbation $\hat{H}'$ within the degenerate subspace of $\hat{H}_0$.

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11. Calculation the probability of interlevel transition for harmonic oscillator and hydrogen atom in external electromagnetic wave by using “golden rule”. Selecton rules.

We consider a quantum system (harmonic oscillator or hydrogen atom) in an external electromagnetic field. The time-dependent perturbation due to the field is:

$$\vec{E} = \vec{E}_0 \cos \omega t, \quad \text{(approximation)}$$

Assuming the field is polarized along the z-axis, the perturbation operator becomes:

$$\hat{H}'(t) = ezE_0 \cos \omega t = \frac{ezE_0}{2}\left(e^{i\omega t} + e^{-i\omega t}\right)$$

The transition probability per unit time from initial state i to final state f is given by Fermi's Golden Rule:

$$w_{i\to f} = \frac{2\pi}{\hbar}\left| f\hat{H}'i \right|^2 \rho(E_f)$$

Using the time-independent part of the perturbation:

$$\hat{h} = \frac{ezE_0}{2}$$

Harmonic Oscillator

For a 1D quantum harmonic oscillator, the selection rule is:

$$\Delta n = \pm 1$$

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The relevant dipole matrix elements are:

$$n \pm 1zn \neq 0$$

Hence, the transition probability becomes:

$$w_{n\to n\pm 1} = \frac{2\pi}{\hbar}\left| n \pm 1\hat{h}n \right|^2 \rho(E_{n\pm 1})$$

Hydrogen Atom

For the hydrogen atom, the dipole selection rules are:

$$\Delta l = \pm 1, \quad \Delta m = 0, \pm 1$$

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The transition matrix element is:

$$n'l'm'\hat{h}nlm = \frac{eE_0}{2} n'l'm'znlm$$

Thus, the transition probability is:

$$w_{i\to f} = \frac{2\pi}{\hbar}\left| \frac{eE_0}{2} fzi \right|^2 \rho(E_f)$$

14. Difference Between Schrödinger and Klein-Gordon Equations

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The time-dependent Schrödinger equation for a free particle:

$$i\hbar\frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2M}\nabla^2\psi$$

Properties:

- First-order in time derivative.

- Describes non-relativistic quantum systems.
- Probabilistic interpretation is valid: $|\psi|^2$ is the probability density.

The Klein-Gordon equation:

$$\frac{1}{c^2}\frac{\partial^2 \psi}{\partial t^2} - \nabla^2 \psi + \frac{m_0^2 c^2}{\hbar^2}\psi = 0$$

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Properties:

- Second-order in both time and spatial derivatives.
- Describes free relativistic scalar (spin-0) particles.
- Derived continuity equation shows ρ is not always positive:

  $$\rho = \frac{i\hbar}{2m_0 c^2}\left(\psi^* \frac{\partial \psi}{\partial t} - \psi \frac{\partial \psi^*}{\partial t}\right)$$

  which invalidates direct probability interpretation.
- Admits negative energy solutions: $E = \pm\sqrt{p^2 c^2 + m_0^2 c^4}$.

Summary

- Schrödinger is non-relativistic; Klein-Gordon is relativistic.
- Schrödinger is first-order in time; Klein-Gordon is second-order.
- Schrödinger allows a clear probability interpretation, Klein-Gordon does not.
- Schrödinger is for electrons in atoms; Klein-Gordon for spin-0 particles in field theory.