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Loengu 14-16

Artur Medri 232924YAFB

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2.

We consider a Hamiltonian of the form:

$$\hat{H} = \hat{H}^{(0)} + \lambda \hat{H}^{(1)}$$

Where:

- $\hat{H}^{(0)}$ is the unperturbed Hamiltonian.
- $\hat{H}^{(1)}$ is the perturbation.
- λ is a small parameter.

The time-independent Schrödinger equation becomes:

$$\hat{H}|\psi_n\rangle = E_n|\psi_n\rangle$$

Suppose the unperturbed energy level $E^{(0)}$ is **d-fold degenerate**, meaning there are d orthonormal eigenstates $\{|\psi_a^{(0)}\rangle\}_{a=1}^{d}$ such that:

$$\hat{H}^{(0)}|\psi_a^{(0)}\rangle = E^{(0)}|\psi_a^{(0)}\rangle$$

To find the first-order corrections, we diagonalize the perturbation $\hat{H}^{(1)}$ **within the degenerate subspace**:

$$H_{ab}^{(1)} = \langle\psi_a^{(0)}|\hat{H}^{(1)}|\psi_b^{(0)}\rangle$$

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We solve the eigenvalue problem:

$$\sum_{b=1}^{d} H_{ab}^{(1)} c_b = E^{(1)} c_a$$

10. Derive and explain! 5

Photon Emission Rate (Transition $n \to m$)

The **Fermi's Golden Rule** gives the transition rate from an initial state $|n\rangle$ to a final state $|m\rangle$ due to a perturbation $\hat{H}^{(1)}(t)$:

$$\Gamma_{n \to m} = \frac{2\pi}{\hbar} |\langle m|\hat{H}^{(1)}|n\rangle|^2 \rho(E_m)$$

Where:

- $\Gamma_{n \to m}$ is the **transition rate** (photons per second).
- $\langle m|\hat{H}^{(1)}|n\rangle$ is the **matrix element** of the perturbation between the states.
- $\rho(E_m)$ is the **density of final states** at energy E_m.

Physical Interpretation

- This formula tells us how likely it is per unit time for a system in state $|n\rangle$ to transition to $|m\rangle$, emitting a photon of energy $\hbar\omega = E_n - E_m$.
- The **number of photons emitted per second** is directly proportional to the square of the transition matrix element and the density of available final states.

15.

1. Schrödinger Equation

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- **Type**: Non-relativistic quantum mechanics
- **Form**:

$$i\hbar\frac{\partial}{\partial t}\psi(\mathbf{r},t)=\hat{H}\psi(\mathbf{r},t)$$

- **Describes**: Particles moving at speeds much less than the speed of light.
- **Wavefunction**: Scalar function ψ
- **Does not include**: Spin or relativistic effects.
- **Limitation**: Cannot describe particles moving near the speed of light or account for antimatter.

2. Dirac Equation

- **Type**: Relativistic quantum mechanics
- **Form**:

How about an explanation of the parameters?

$$(i\hbar\gamma^\mu\partial_\mu - mc)\psi = 0$$

- **Describes**: Particles like electrons at relativistic speeds.
- **Wavefunction**: Four-component spinor (accounts for spin-½ particles).
- **Includes**:
  - Special relativity
  - Spin naturally
  - Predicts antimatter (e.g., positrons)
- **Reduces to Schrödinger equation** in the low-energy limit.