

Kodutöö 4

Madis Unt

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- 1 **5. Is it possible to measure simultaneously square of angular momentum and its z projections of angular momentum (L^2 and L_z)? Why? Proof.**

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Taking the L^2 operator we can break it down to its components L_x , L_y and L_z , which if added up make L^2 . These components satisfy the standard commutation relations. So taking our problem's operators L^2 and L_z and commute them we get $L_x + L_y + L_z \cdot L_z$, breaking this down we get $\text{commute}(L_x \cdot L_z) + \text{commute}(L_y \cdot L_z) + \text{commute}(L_z \cdot L_z)$. Turning those into the commutation relations and simplifying a bit we get $i\hbar \text{covered}(L_x \cdot L_y + L_y \cdot L_x)$ for $\text{commute}(L_x \cdot L_z)$. For $\text{commute}(L_y \cdot L_z)$ it's $-i\hbar \text{covered}(L_x \cdot L_y + L_y \cdot L_x)$ and $\text{commute}(L_z \cdot L_z)$ is zero. Now summing all the parts we can see that it's zero, meaning that the commute itself is zero. Meaning yes it is possible because they are independent of each other.

- 2 **17. A disk with a radius of 1 mm and a mass of 1 mg rotates around an axis perpendicular to its plane with a frequency of 1 Hz. Calculate the minimal possible value for angle between vector L and z-axis.**

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We have a radius $r = 1 \text{ mm} = 0,001 \text{ m}$, a mass of $m = 1 \text{ mg} = 0,001 \text{ kg}$ and a frequency $f = 1 \text{ Hz} \Rightarrow \omega = 2\pi \text{ rad/s}$ around our z axis. $L = I\omega = (1/2)Mr^2 \cdot 2\pi = Mr^2\pi = \pi \cdot 10^{-12}$. quantum number $l = L/\hbar = 2,98 \cdot 10^{-22}$. $\alpha_{\min} = \sqrt{1/l} = 5,78 \cdot 10^{-12} \text{ rad}$.

?

3 20. Obtain the equation for radial part of wave function:

0

Due to time constraints and the formula being very long, I could not finish this in time. I will be sending a version with hopefully a correct answer in this space but for now for me to submit this on time I will leave it.

4 31. Calculate the possible maximum number of electrons in 4p and 2d orbitals for hydrogen atom.

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So we have 2 orbitals 4p and 2d with principal quantum numbers $n_1=4$ and $n_2=2$. The azimuthal quantum numbers are $l_1=3$ and $l_2=1$. Taking first the first orbital and its levels for the azimuthal numbers are: $l=0, 1, 2$, we care about the p orbital so our l is $l=1$. thus out magnetic quantum number is $m = [-l, l] = -1, 0, 1$. Because each orbital can have 2 electrons, so the maximum number of electrons we can have for 4p is $3 \times 2 = 6$. Moving on to 2d. Since $l=1$ means that we only have levels: $l=0$ and $l=1$, but no d level, meaning we don't have a 2d orbital.

5 42. Can you prove the following expression (page 127): "In the first order approximation of lambda coefficient a_n must satisfy $a_n + (a_n)^* = 0$ ".

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So the statement refers to a coefficient a_n that in our case is undetermined and needs to be determined. The determination of it comes for the normalization of the first order wave function ($\int (\psi_0 + \lambda \sum_m a_m \psi_m)^* (\psi_0 + \lambda \sum_m a_m \psi_m) dr$). The part is imaginary and $a_n=0$ is taken for simplicity.