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Loengud 10-13

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Question 4

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Is it possible to measure simultaneously the x, y, and z projections of angular momentum? Why? Give proof.

Answer

No, it is not possible to measure simultaneously the x, y, and z components of angular momentum.

In quantum mechanics, observables are represented by Hermitian operators, and two observables can be simultaneously measured only if their operators commute. The components of angular momentum are represented by operators $\hat{L}_x, \hat{L}_y, \hat{L}_z$, which satisfy the following commutation relations:

$$[\hat{L}_x, \hat{L}_y] = i\hbar \hat{L}_z, \quad [\hat{L}_y, \hat{L}_z] = i\hbar \hat{L}_x, \quad [\hat{L}_z, \hat{L}_x] = i\hbar \hat{L}_y$$

Since these commutators are nonzero, the operators do not commute. Therefore, the uncertainty principle applies, and it is impossible to simultaneously measure all three components of angular momentum with arbitrary precision.

However, it is possible to measure $\hat{L}^2 = \hat{L}_x^2 + \hat{L}_y^2 + \hat{L}_z^2$ and one component, typically $\hat{L}_z$, simultaneously because:

$$[\hat{L}^2, \hat{L}_z] = 0$$

Conclusion: It is not possible to measure all three projections $\hat{L}_x, \hat{L}_y, \hat{L}_z$ simultaneously because their operators do not commute.

Question 19

How to calculate in quantum mechanics the kinetic energy of a rotating body with moment of inertia I? If the body is an electron moving around a nucleus in orbit with radius $r = 10^{-10}$ m, calculate the minimum possible non-zero velocity.

Answer

is it necessary to use the relativistic corrections?

In quantum mechanics, rotational kinetic energy is:

$$E = \frac{L^2}{2I}$$

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The smallest non-zero angular momentum is $L = \sqrt{2}\hbar$ (for $\ell = 1$).

For an electron orbiting with radius r, the moment of inertia is $I = m_e r^2$. Substituting:

$$E = \frac{2\hbar^2}{2m_e r^2} = \frac{\hbar^2}{m_e r^2}$$

Using $E = \frac{1}{2}m_e v^2$, solve for v:

$$v = \sqrt{\frac{2\hbar^2}{m_e^2 r^2}}$$

Insert values:

$$\hbar = 1.055 \times 10^{-34}, \quad m_e = 9.11 \times 10^{-31}, \quad r = 10^{-10}$$

$$v \approx \sqrt{\frac{2 \cdot (1.055 \times 10^{-34})^2}{(9.11 \times 10^{-31})^2 \cdot (10^{-10})^2}} \approx 2.7 \times 10^6 \, \mathrm{m/s}$$

Final answer:

$$\boxed{v_{\min} \approx 2.7 \times 10^6 \, \mathrm{m/s}}$$

Question 22

What does the equation for the radial wave function look like if we assume that the electron is an uncharged particle and moves around the nucleus in an orbit with a fixed radius ($r = \text{const}$)?

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Answer

If the electron is uncharged, there is no Coulomb potential ($V(r) = 0$), and if the radius is fixed ($r = \text{const}$), the radial motion is frozen. The electron behaves like a particle constrained to a spherical shell.

The time-independent Schrödinger equation reduces to the angular part only. The radial wave function becomes a delta-like constraint at fixed radius:

$$R(r) \propto \delta(r - r_0)$$

Since the motion is restricted to a constant radius, there is no radial kinetic or potential energy. The only relevant solutions are spherical harmonics $Y_{\ell m}(\theta, \phi)$, and the energy depends only on angular momentum:

$$E_\ell = \frac{\hbar^2 \ell(\ell+1)}{2mr_0^2}$$

Conclusion: The radial wave function is sharply peaked at $r = r_0$, i.e.

$$\boxed{R(r) \propto \delta(r - r_0)}$$

Question 33

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What does the electron configuration look like for O, Al, and Li atoms? Why?

Answer

The electron configuration describes how electrons occupy atomic orbitals, following the Aufbau principle, Pauli exclusion principle, and Hund's rule.

1. Lithium (Li, $Z = 3$):

$$\boxed{\text{Li: } 1s^2\, 2s^1}$$

Two electrons fill the $1s$ orbital, and the third enters the $2s$ orbital.

2. Oxygen (O, $Z = 8$):

$$\boxed{\text{O: } 1s^2\, 2s^2\, 2p^4}$$

After filling $1s$ and $2s$, the next 4 electrons partially fill the $2p$ orbitals.

3. Aluminum (Al, $Z = 13$):

$$\boxed{\text{Al: } 1s^2\, 2s^2\, 2p^6\, 3s^2\, 3p^1}$$

After filling the first and second shells completely, one electron enters the $3p$ orbital.

Why: Electrons fill orbitals in order of increasing energy. The given configurations minimize the total energy while obeying quantum mechanical rules.

Question 44

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Can you prove that for a harmonic oscillator in a constant force field (see p. 129) the first-order energy correction is zero, i.e. $E_n^{(1)} = \langle n|\hat{H}'|n\rangle = 0$?

Answer

The perturbation due to a constant force F is:

$$\hat{H}' = -F\hat{x}$$

First-order energy correction in perturbation theory:

$$E_n^{(1)} = \langle n|\hat{H}'|n\rangle = -F\langle n|\hat{x}|n\rangle$$

In a harmonic oscillator, the expectation value of position in any stationary state is zero:

$$\langle n|\hat{x}|n\rangle = 0$$

Because the harmonic oscillator eigenstates are either symmetric or anti-symmetric, and $\hat{x}$ is an odd operator, the integral over an odd function yields zero.

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Conclusion:

$$\boxed{E_n^{(1)} = 0}$$

The first-order energy shift vanishes for all states n.