I. Angular momentum (general properties)

1. Derive classical expressions for L_x, L_y and L_z projections for **angular momentum vector**.

2. Derive expressions for the projections of the angular momentum operators $\hat{L}_x = -i\hbar\left(y\frac{\partial}{\partial z} - z\frac{\partial}{\partial y}\right)$, $\hat{L}_y = -i\hbar\left(z\frac{\partial}{\partial x} - x\frac{\partial}{\partial z}\right)$, $\hat{L}_z = -i\hbar\left(x\frac{\partial}{\partial y} - y\frac{\partial}{\partial x}\right)$.

3. Derive the commutation relation for **x** and **y** projections of angular momentum operator $\left[\hat{L}_x, \hat{L}_y\right] = i\hbar\,\hat{L}_z$. How this equation related to Heisenberg uncertainty principle?

4. Is it possible measure simultaneously the **x, y** and **z** projections of angular momentum? Why? Proof.

5. Is it possible measure simultaneously square of angular momentum and its **z** projections of angular momentum ($\hat{L}^2$ and $\hat{L}_z$)? Why? Proof.

6. Is it possible measure simultaneously square of angular momentum and its **x** projection ($\hat{L}^2$ and $\hat{L}_x$)? Why? Proof.

7. Is it possible measure simultaneously absolute value of angular momentum $|\vec{L}|$ and its **x** projection? Why? Proof.

8. Can you proof the expression $\left[\hat{H}, \hat{L}^2\right] = 0$. What does it means from a physical point of view? How this equation related to Heisenberg uncertainty principle? Here $\hat{H} = -\frac{\hbar^2}{2m}\Delta$.

9. Can you proof the expression $\left[\hat{H}, \hat{L}_z\right] = 0$. What does it means from physical point of view?How this equation related to Heisenberg uncertainty principle? Here $\hat{H} = -\frac{\hbar^2}{2m}\Delta$.

10. Find a solution to the eigenvalue problem for the operator $\hat{L}_z$ (Derive!).

11. Find a solution to the eigenvalue problem for the operator $\hat{L}^2$ (Derive!).

12. Derive that relation between orbital and magnetic quantum numbers $m = -l, \ldots, 0, \ldots +l$.

II. Angular momentum (practical application)

13. Write an expression for the **x, y** and **z** projection of circular frequency operators $\hat{\omega}_x, \hat{\omega}_z, \hat{\omega}_z$ (The rotating body is a **sphere** of mass **M** and radius **R**).

14. How can the angle between the angular momentum vector $\vec{L}$ and the **z**-axis be calculated in quantum mechanics? Calculate the allowed **possible** values of this angle for ***2p*** **-** orbital.

15. A disk with a radius of **1 mm** and a mass of **1 mg** rotates around an axis perpendicular to its plane with a frequency of **1 Hz**. Calculate the value of orbital quantum number ***l*** for this spinning disk.

16. How in quantum mechanics can be calculated the angle between the angular momentum vector $\vec{L}$ and the **z**-axis? Calculate the values of this angle for **magnetic quantum numbers m=-2 and +1** for ***3d*** orbitals .

17. A disk with a radius of **1 mm** and a mass of **1 mg** rotates around an axis perpendicular to its plane with a frequency of 1 Hz. Calculate the minimal possible value for angle between vector $\vec{L}$ and **z**-axis.

18. How to calculate in quantum mechanics the kinetic energy of a rotating body with the moment of inertia **I** ? Calculate the velocity of an electron moving around the nucleus in an orbit with a radius 10^{-10} m. The orbital quantum number of electron equal to $l=5$ (NB! Electron is a point particle).

19. How to calculate in quantum mechanics the kinetic energy of a rotating body with the moment of inertia **I** ? If body is an electron moving around nucleus in orbit with radius 10^{-10} m. Calculate the minimum **possible non zero** value of velocity of an electron (NB! Electron is a point particle).

III. Hydrogen atom

20. Obtain the equation for radial part of wave function:

$$-\frac{\hbar^2}{2M}\left[\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dR(r)}{dr}\right)\right]+\left[\frac{\hbar^2 l(l+1)}{2M\,r^2}+U(r)\right]R_{nl}(r)=E\,R_{nl}(r)$$

from the general Schrödinger equation for hydrogen atom

$$-\frac{\hbar^2}{2M}\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right)-\frac{1}{\hbar^2 r^2}\,\hat{\vec{L}}^2_{\theta\,\phi}-\frac{2M}{\hbar^2}\,U(r)\right]\psi_{nlm}(\vec{r})=E\,\psi_{nlm}(\vec{r})\ .$$

here $\psi_{nlm}=R_{nl}(r)\,Y_{lm}(\theta,\phi)$

21. What does the equation for the radial wave function look like if we assume that the electron is an uncharged particle?

22. What does the equation for the radial wave function look like if we assume that the electron is an uncharged particle and moves around the nucleus in an orbit with a fixed radius (r=const)?

23. Write the radial part of wave function R_{nl} for quantum numbers n=3 and l=2.

24. How looks like the radial part of wave function for 3s , 2d and 1f states?

25. How can be calculate the ionization energy of an electron from the ground state?

26. How can be calculate the ionization energy of an electron from the states 3s and 4p?

27. The total energy of electron in hydrogen atom can be calculated as follows : $E_n=-R\hbar\frac{1}{n^2}$. But how can be calculate separately the kinetic and potential energy of an electron in hydrogen atom for principal quantum number n=3 ?
NB! The classical relation between the potential and kinetic energies for electron in hydrogen atom can be used.

28. Show that (for hydrogen atom) the number of sublevels (degree of degeneration) for energy level with fixed value of principle quantum number ***n*** can be calculated with formula ***n²***. Does this calculation take into account the spin effect?

IV. The spectroscopic properties of hydrogen like atoms.

29. Calculate the photon wavelength required to ionize a hydrogen atom from the ground state.

30. Calculate the possible maximum number of electrons in 1p and 3f orbitals for hydrogen atom.

31. Calculate the possible maximum number of electrons in 4p and 2d orbitals for hydrogen atom.

32. Electron configuration for B atom is looks like so: $1s^2 2s^2 2p^1$. What does it means? Describe all numbers in the configuration description.

33. How looks like the electron configuration for O, Al and Li atoms? Why?

34. How looks like the electron configuration for N ,Cl and K atoms? Why?

35. How looks like the electron configuration for Cu atom and Cu^{2-} ion? Why?

36. How looks like the electron configuration for Li, Na and K atoms? Why? What do all these materials have in common (in terms of physical properties)? How does this relate to the configuration of the electrons?

37. How looks like the electron configuration for Ne and Kr atoms? Why? What do all these materials have in common (in terms of physical properties)? How does this relate to the configuration of the electrons?

38. Why the vector of angular momentum and magnetic moment of electron are have an opposite directions?

39. What does the eigenvalue problem look like for the operator of square of the orbital magnetic moment $\hat{\mu}_l^2$ of an electron in a hydrogen atom? Present the wavefunction and eigenvalues of this operator.

V. The perturbation theory (non degenerate case)

40. Write the Schrodinger equation for **third order** approximation of perturbation theory.

41. Write the Schrodinger equation for **zero order** approximation of perturbation theory.

42. Can you prove the following expression (page 127): "In the first order approximation of λ coefficient a_n^1 must satisfy $a_n^1 + (a_n^1)^* = 0$ ".

43. Can you prove the following equation **for second order approximation** (page 129): $\sum_{k \neq n} |a_k^1|^2 + ((a_n^2)^* + a_n^2) = 0$

44. Can you prove that for **harmonic oscillator in constant force field** (page 129) "...the first order energy correction is equal to zero $E_n^1 = H'_{nn} = 0$ ".

45. Why for the second-order energy correction of a harmonic oscillator in a constant force field (p. 129) we need to take into account only two terms of the sum n, n + 1 and n, n-1

$$E_n^2 = \sum_{k \neq n} \frac{|H'_{nk}|^2}{E_n^0 - E_k^0} = \frac{|H'_{n,\,n+1}|^2}{E_n^0 - E_{n+1}^0} + \frac{|H'_{n,\,n-1}|^2}{E_n^0 - E_{n-1}^0} = \frac{F^2}{\hbar\omega}(-|x_{n,n+1}|^2 + |x_{n,n-1}|^2) = -\frac{F^2}{2M\omega^2}.$$

46. Can you prove that for anharmonic oscillator the first order energy correction for cubic term must be equal zero(page 130)?